# High-beta Cavity Design

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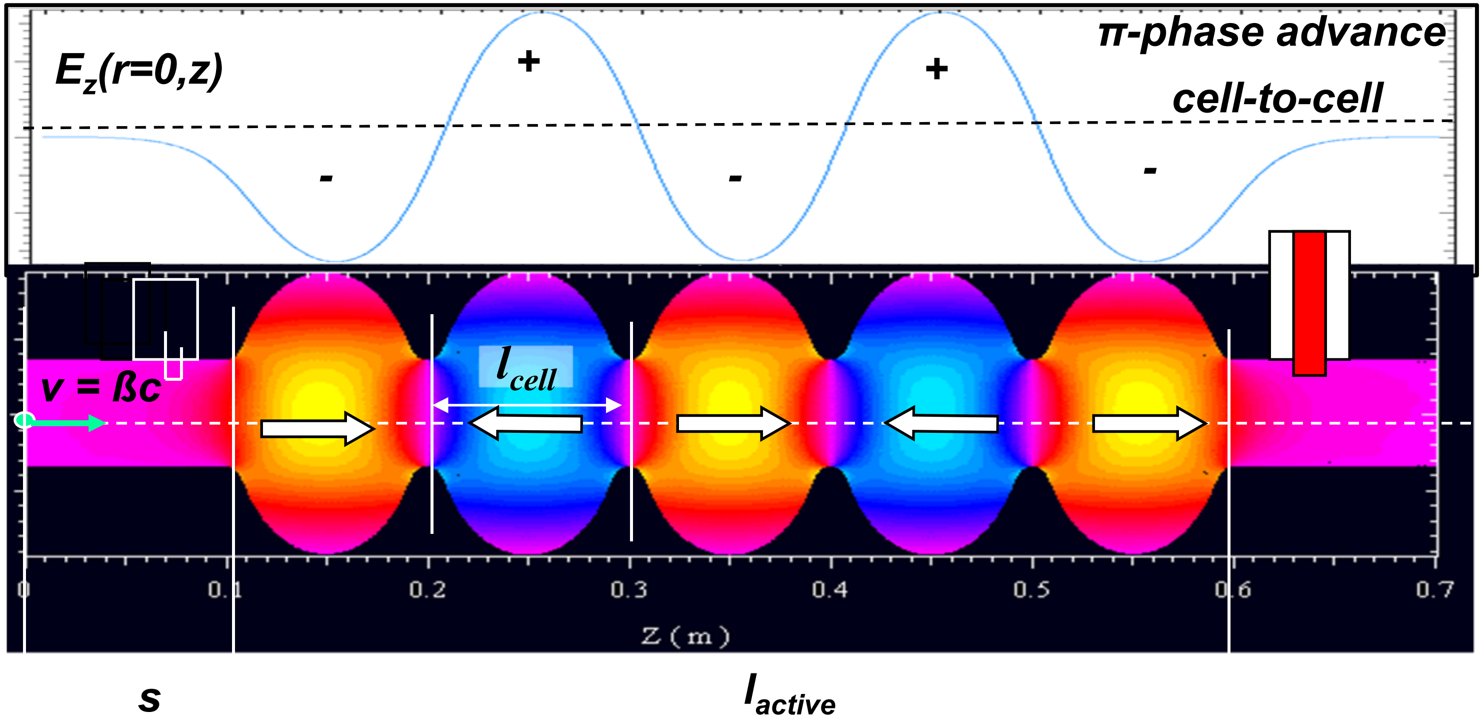
1. SW Operation with SRF RF Cavity
2. Pill Box Cavity
3. Figure of merits of SRF Cavity Design
4. Criteria for Multi-Cell Structures
5. Example of SRF High-beta Cavities

**Acknowledgement:**
In this lecture, I used many slides by Dr. J.Sektuwitz@ DESY, which he made nice lecture in China. I would like to thank for him very much.

# 1. Sanding Wave (SW) Operation and Standing Wave in SRF Cavity

# SW Scheme in SRF Cavity Operation




TM$_{010}$
π-mode

**Standing wave (CW) is used in SRF cavity acceleration !**

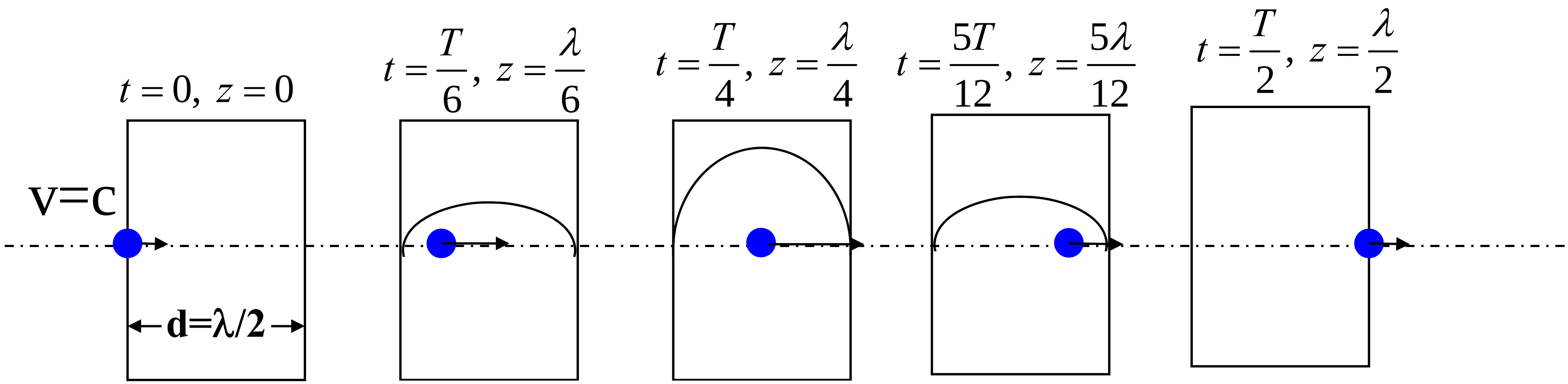
***Synchronic acceleration and max of $(R/Q)_{acc} \leftrightarrow l_{active} = Nl_{cell} = Nc\beta/(2f)$ and the injection takes place at an optimum phase $\varphi_{opt}$ which ensures that particles will arrive at the mid-plane of the first cell when $E_{acc}$ reaches its maximum (+q passing to the right) or minimum (-q passing to the right).***

$$L_{cell} = \frac{\beta \cdot \lambda}{2}, \qquad \lambda = \frac{c}{f}$$

β=v/c

for electron β=1, proton β < 1

# Transit Time Factor Due to SW Operation





$$V=\left|\int_0^d E_z(r=0,z)e^{i\omega t}dz\right|=\left|\int_0^d E_z(r=0,z)e^{i\omega\frac{z}{c}}dz\right|=E_0\left|\int_0^d e^{i\omega\frac{z}{c}}dz\right|=E_0 d\,\frac{\sin\left(\frac{\omega d}{2c}\right)}{\frac{\omega d}{2c}}=E_0 d\cdot T$$

$$z=c\cdot t$$

$T$ : Transit time factor

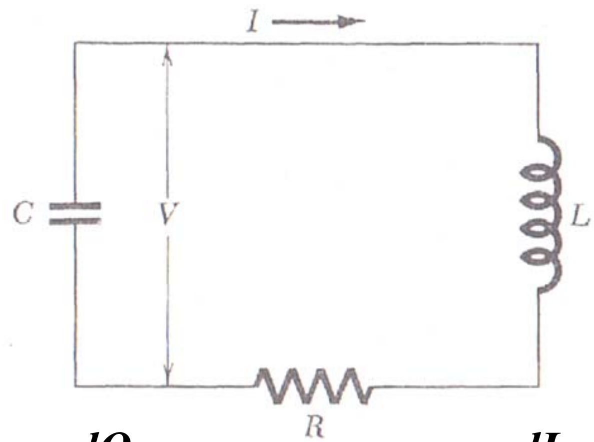
$T=\frac{2}{\pi}=0.637$ (for Pill Box Cavity)

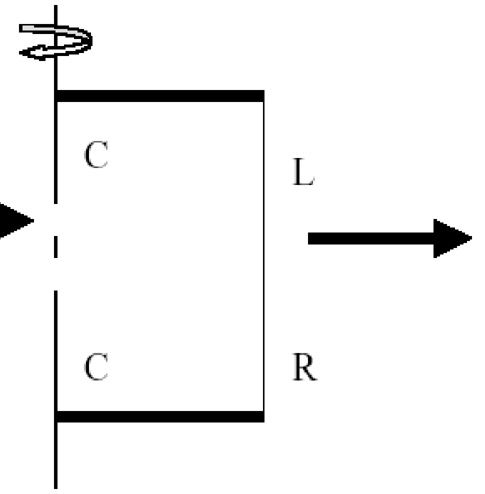
$Eacc\equiv\frac{V}{d}=E_0 T$

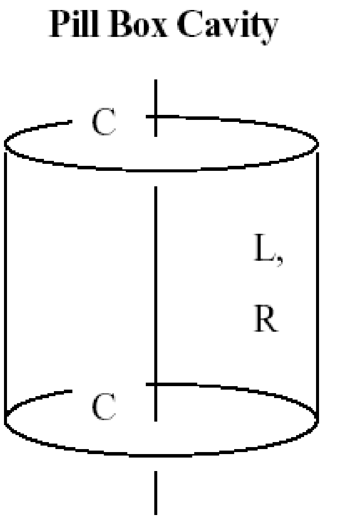
**Acceleration efficiency is automatically reduced by ~ 40% in the SW scheme.**

# 2. Pill Box Cavity

# LC Equivalent Circuit of Cavity




$$I = -\frac{dQ}{dt}, \quad Q = CV, \quad V = L\frac{dI}{dt} + RI$$

$$\frac{d^2V}{dt} + \left(\frac{R}{L}\right)\frac{dV}{dt} + \left(\frac{1}{LC}\right) = 0, \quad V(t) = V_O \exp(-\alpha + i\omega)t$$

$$(-\alpha + i\omega)^2 + (-\alpha + i\omega)\left(\frac{R}{L}\right) + \left(\frac{1}{LC}\right) = 0,$$

$$\alpha = \frac{R}{2L}, \quad \omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

$$R << L, \quad \omega_O^2 = \frac{1}{LC} \Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

## Q-value of the circuit

$$Q \equiv \omega \cdot \frac{\text{Stored Energy}}{\text{Powerloss/sec}} = \omega \cdot \frac{P(t)}{dP(t)/dt} = \omega \cdot \frac{L}{R}$$

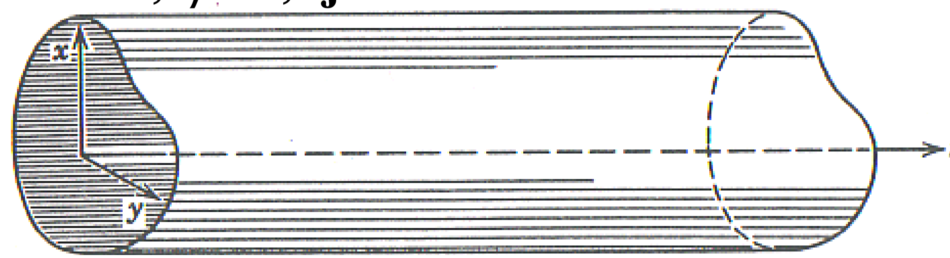
$$= \frac{\omega}{2\alpha}$$

**Q : proportional to 1/R**

# Electro-magnetic field in a waveguide

**Maxwell equations in a waveguide**

$$\nabla\times\mathbf{E}=i\frac{\omega}{c}\mathbf{B},\ \nabla\cdot\mathbf{B}=0,\ \nabla\times\mathbf{B}=-i\mu\varepsilon\frac{\omega}{c}\mathbf{E},\ \nabla\cdot\mathbf{E}=0,\ \rho=0,\ \mathbf{j}=0$$




$$\left(\nabla^2+\mu\varepsilon\frac{\omega^2}{c^2}\right)\begin{Bmatrix}\mathbf{E}\\ \mathbf{B}\end{Bmatrix}=0,$$

$$\mathbf{E}(x,y,z,t)=\mathbf{E}(x,y)\exp(\pm ikz-i\omega t),\ k\text{: wavevector},$$

$$\mathbf{B}(x,y,x,t)=\mathbf{B}(x,y)\exp(\pm ikz-i\omega t),$$

$$\left[\nabla_t^2+(\varepsilon\mu\frac{\omega^2}{c^2}-k^2\right]\begin{Bmatrix}\mathbf{E}\\ \mathbf{B}\end{Bmatrix}=0,\quad \nabla_t^2\equiv\nabla^2-\frac{\partial^2}{\partial^2 z},\quad \mathbf{E}=E_z\mathbf{e}_z+\mathbf{E}_t,\quad \mathbf{B}=B_z\mathbf{e}_z+\mathbf{B}_t$$

$$\mathbf{B}_t=\frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2}-k^2\right)}\left[\nabla_t\left(\frac{\partial B_z}{\partial z}\right)+i\varepsilon\mu\frac{\omega}{c}\mathbf{e}_z\times\nabla_t E_z\right],$$

$$\mathbf{E}_t=\frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2}-k^2\right)}\left[\nabla_t\left(\frac{\partial E_z}{\partial z}\right)-i\frac{\omega}{c}\mathbf{e}_z\times\nabla_t B_z\right]$$

**Homework I.**
**Lead this formula.**

$TM$ mode: $B_z=0,\ E_z\neq 0$

$TE$ mode : $B_z\neq 0,\ E_z=0$

# TM- Modes

TM-mode : $B_z = 0,\ E_z \neq 0$ ⟹ Can accelerate beam

Beam

$$\mathbf{B}_t = \frac{i\varepsilon\mu \frac{\omega}{c}}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}[\mathbf{e}_z \times \nabla_t E_z],$$

$$\mathbf{E}_t = \frac{1}{\left(\varepsilon\mu\frac{\omega^2}{c^2} - k^2\right)}\nabla_t\left(\frac{\partial E_z}{\partial z}\right),$$

$$\left[\nabla_t^2 E_z + (\varepsilon\mu\frac{\omega^2}{c^2} - k^2)\right]E_z = 0,$$

⟹ **Solve the eigenvalue problem, get k and Ez**

**Boundary condition** $E_z|_S = 0$ **(** $\because$ $\mathbf{n}\times\mathbf{E} = 0$ **on the surface of perfect conducto**

$\frac{\partial B_z}{\partial n}|_S = 0$ **(** $\because$ $\mathbf{n}\cdot\mathbf{B} = 0$ **on the surface,**

**but automatically satisfied by the TM mode condition)**

**Similar for TE modes, which kicks beam……**

# Eigen-value problem

$\psi(x,y) = E_z(x,y)$ **for TM- mode or** $B_z(x,y)$ **for TE- mode**

$$\left(\vec{\nabla}_t^2 + \gamma^2\right)\psi = 0,\ \ \psi\big|_S = 0 \text{ (for TM-Modes) or } \frac{\partial}{\partial n}\psi\big|_S = 0 \text{ (for TE-Modes)}$$

$$\gamma^2 = \varepsilon\mu \cdot \frac{\omega^2}{c^2} - k^2 \geq 0$$

From the boundary condition,

$$\gamma^2 = \gamma_\lambda^2\ ,\ \psi = \psi_\lambda \quad (\lambda = 1, 2, \cdots) \Longrightarrow \quad k_\lambda^2 = \varepsilon\mu\frac{\omega^2}{c^2} - \gamma_\lambda^2$$

If $\omega < c\dfrac{\gamma_\lambda}{\sqrt{\varepsilon\mu}}$, then $k_\lambda$ is an imaginal number.

The wave is damped/trapped in the waveguide, if surface resistance large/small.

$$\omega_\lambda = c\frac{\gamma_\lambda}{\sqrt{\varepsilon\mu}} \cdots \text{cutoff frequency}$$

When $\omega \geq \omega_\lambda$ , wave number $k_\lambda$ is a real number,

then the wave can propagate into the waveguide.

$TM_{m,n,p}$ – mode

$$E_z = E_o \cos(kz) J_m(\frac{\rho_{m,n}}{a} r) \exp(-im\theta), \qquad B_z = 0$$

$$E_r = \frac{iE_0 p\pi}{\gamma_{m,n,p}} \cos(\frac{p\pi}{d} z) \frac{\partial J_m(\rho)}{\partial \rho} \exp(-im\theta), \qquad B_r = -\frac{E_0 m\varepsilon\mu\omega_{m,n,p}}{\gamma_{\mathrm{m,n,p}}} \cos(kz) J_m(\frac{\rho_{m,n}}{a} r) \exp(-im\theta)$$

$$E_\theta = \frac{E_0 m p\pi}{\gamma_{m,n,p}^2 dc} \cos(\frac{p\pi}{d} z) J_m(\frac{\rho_{m,n}}{a} r) \exp(-im\theta), \quad B_\theta = \frac{iE_0 \varepsilon\mu\omega_{m,n,p}}{\gamma_{m,n,p} c} \cos(kz) \exp(-im\theta) \frac{\partial J_m(\rho)}{\partial \rho}$$

resonace frequency $\qquad \omega_{m,n,p} \Rightarrow f_{m,n,p} = \frac{c}{\sqrt{\varepsilon\mu}} \sqrt{\frac{\rho_{m,n}^2}{a^2} + \frac{p^2\pi^2}{d^2}}$

**TE$_{mnp}$ Modes**

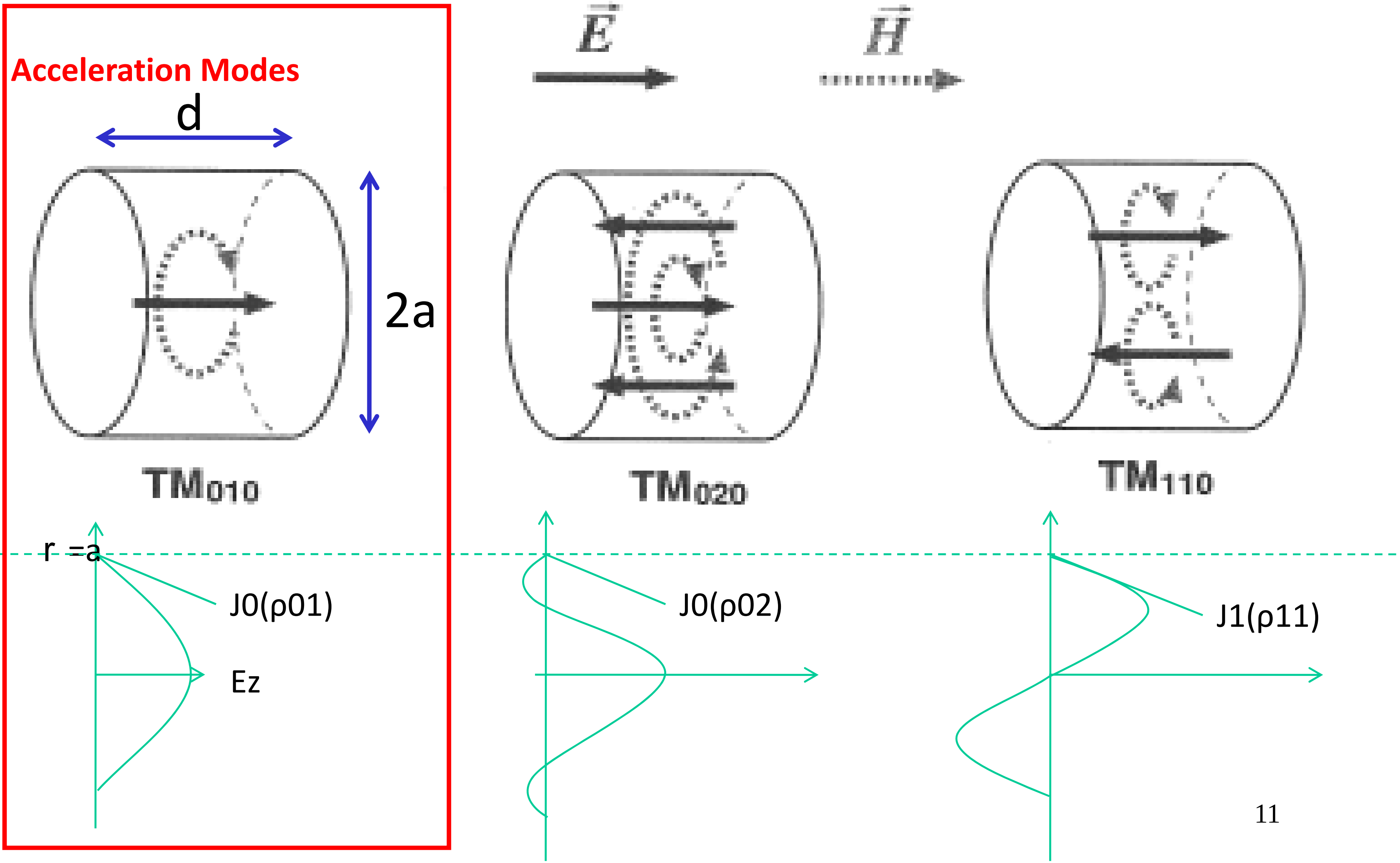
$$E_z = 0 \qquad H_z = E_0 J_m\left(\frac{\rho'_{mn}}{a} r\right) \cos(m\theta) \sin(\frac{p\pi z}{d})$$

$$E_r = \frac{i\omega\varepsilon}{k^2} \frac{m}{r} E_0 J_m\left(\frac{\rho'_{mn}}{a} r\right) \sin(m\theta) \sin(\frac{p\pi z}{d}) \qquad H_r = \frac{1}{k^2} \frac{p\pi}{d} E_0 J_m\left(\frac{\rho'_{mn}}{a} r\right) \cos(m\theta) \cos(\frac{p\pi z}{d})$$

$$E_\theta = \frac{i\omega\varepsilon}{k^2} \frac{\rho'_{mn}}{a} E_0 J_m\left(\frac{\rho'_{mn}}{a} r\right) \cos(m\theta) \sin(\frac{p\pi z}{d}) \qquad H_\theta = -\frac{1}{k^2} \frac{p\pi}{d} \frac{m}{r} E_0 J_m\left(\frac{\rho'_{mn}}{a} r\right) \sin(m\theta) \cos(\frac{p\pi z}{d})$$

Resonance frequency $\qquad \omega^2 \varepsilon_0 \mu_0 = \left(\frac{\rho'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2 \Rightarrow f = \frac{c}{2\pi} \sqrt{\left(\frac{\rho'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$

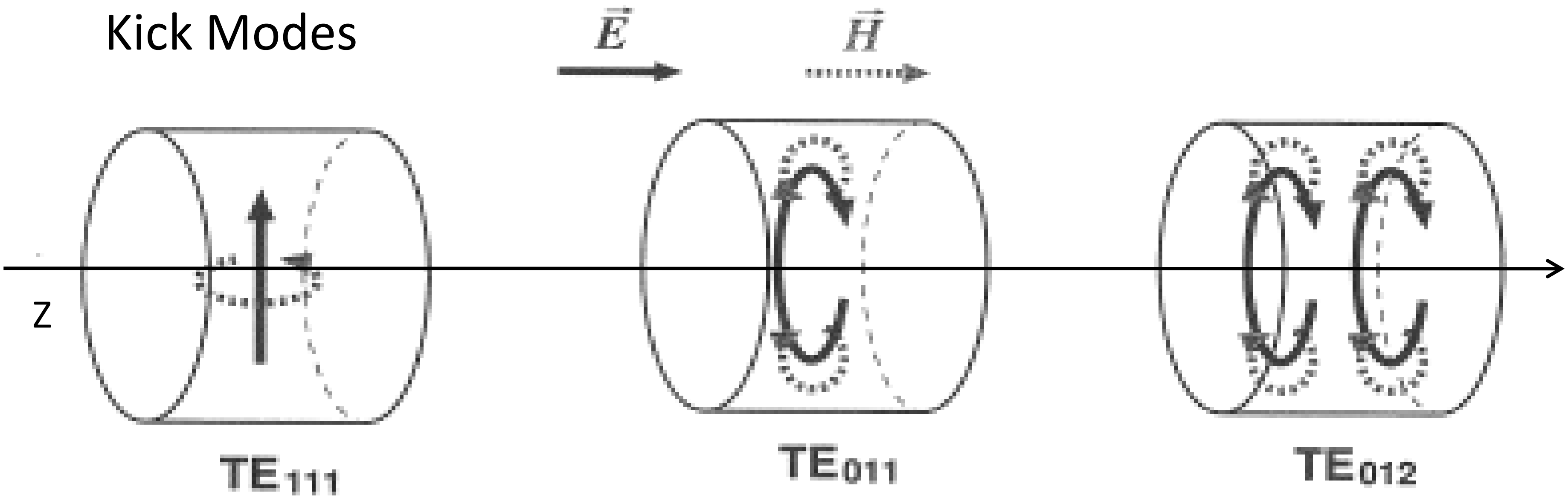
# $TM_{mnp}$ Modes



Acceleration Modes
d
2a
E
H
TM010
TM020
TM110
r =a
J0(ρ01)
Ez
J0(ρ02)
J1(ρ11)

# $TE_{mnp}$ Modes

Kick Modes



# 4. Figure of merits of  Cavity Design

# Figure of Merits of The RF Cavity Design

| | Items | Notation |
|---|---|---|
| 1) | Surface resistance | $R_S$ [Ω] |
| 2) | RF dissipation | $P_{loss}$[W] |
| 3) | Acceleration gradient | $V$[V] |
| 3) | Unloaded Q | $Q_O$ |
| 4) | Geometrical factor | Γ[Ω] |
| 5) | Shunt Impedance | Rsh[Ω] |
| 6) | R over Q | (R/Q) [Ω] |
| 7) | Acceleration gradient | Eacc[V/m] |
| 8) | Ratio of the surface peak electric field vs Eacc | Ep/Eacc [Oe/(MV/m) |
| 9) | Ratio of the surface magnetic field vs Eacc | Hp/Eacc |
| 10) | HOM loss factor | $\kappa_{//}$ , $\kappa_{\perp}$ |
| 11) | Field flatness factor | $a_{ff}$ |
| 12) | Cell to cell coupling | $k_{CC}$ |

# Surface Resistance

**Maxwell Equations.**

$$\nabla\cdot\vec{B}=0,\ \nabla\times\vec{E}+\mu\frac{\partial\vec{H}}{\partial t}=0$$

$$\nabla\cdot\vec{D}=0,\ \nabla\times\vec{H}-\varepsilon\frac{\partial\vec{E}}{\partial t}-\sigma\vec{E}=0$$

$$\vec{J}=\sigma\vec{E}\quad\text{(Ohm's Law)}$$

$$\vec{E}(\vec{x},t)=\vec{E}_{\ell}(\vec{x},t)+\vec{E}_{t}(\vec{x},t),$$

$$\vec{H}(\vec{x},t)=\vec{H}_{\ell}(\vec{x},t)+\vec{H}_{t}(\vec{x},t)$$

For the transvers,

Plane wave : $\vec{E}_t(\vec{x},t)=\vec{E}_t(0)\cdot\exp(i\vec{k}\cdot\vec{x}-\omega t)$

$$\vec{H}_t(x,t)=\frac{1}{\mu\omega}[\vec{k}\times\vec{E}_t(\vec{x},t)],$$

$$[\,k^2-(\varepsilon\mu\omega^2+i\mu\omega\sigma)\,]\begin{Bmatrix}\vec{E}_t(\vec{x},t)\\ \vec{H}_t(\vec{x},t)\end{Bmatrix}=0$$

**Rs: Surface resistance**

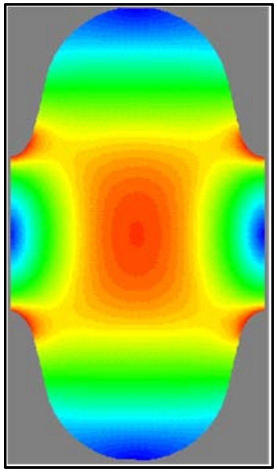
## Surface Impedance

$$Z\equiv R_s+iX_s\equiv\left.\frac{E_t}{H_t}\right|_{Surface}=\frac{\mu\omega}{k}$$

**Homework II**
**Lead the formula of Rs**
**for good electric conductor**

$$\mathbf{R_s}=\sqrt{\frac{\mu\omega}{2\sigma}}=\frac{1}{\sigma}\sqrt{\frac{\mu\sigma\omega}{2}}=\frac{1}{\sigma\delta}\qquad P_{loss}=\frac{1}{2}R_S\cdot\int_S H_S{}^2\,dS$$

# Charismatic RF Parameter of Cavity



*U ≡ stored energy, Ploss:Surface RF Heating*

***Unloaded $Q_O$***

$$U = \frac{1}{2}\cdot\varepsilon\int_V E^2 dV = \frac{1}{2}\cdot\mu\int_V H^2 dV$$

$$P_{loss} = \frac{1}{2}\cdot R_S\int_S H_S^2 dS$$

$$Q_O \equiv \frac{\omega U}{P_{loss}}$$

For acceleration mode (TM010), the higher Qo is better.

***Acceleration Voltage*** $V[\mathrm{V}] = \int_0^{L_{eff}} Edz$ on the beam axis.

***Shunt Impedance***

$$R_{sh}[\Omega] \equiv \frac{V^2}{P_{loss}}$$



This means the efficiency of the acceleration.

# Charismatic RF Parameter of Cavity, continued.

**(R/Q)**

$$(R/Q) \equiv (R_{sh}/Q_O) = \frac{V^2}{\omega U}$$

This means how much energy is concentrated on beam axis.
This does not depend on material but only cavity shape.
This means the goodness of the cavity shape.

**Geometrical factor**

$$\Gamma[\Omega] \equiv Q_O \cdot R_S = \frac{\omega \int_V H^2 dV}{\frac{1}{2}\int_S H_S^2 dS}$$

When you know the $\Gamma$ using a computer code, you can calculate the surface resistance $R_S$ from the measured $Q_O$ value.

$\Gamma$ is about 270Ω for β=1 cavities with elliptical or spherical shape.

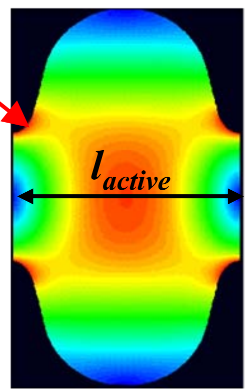
# Charismatic RF Parameter of Cavity

**Gradient**

$$E_{acc} = \frac{\sqrt{(R/Q)}}{L_{eff}} \cdot \sqrt{P_{loss} \cdot Q_O} = Z \cdot \sqrt{P_{loss} \cdot Q_O}$$

**$E_{peak}$ on the surface**

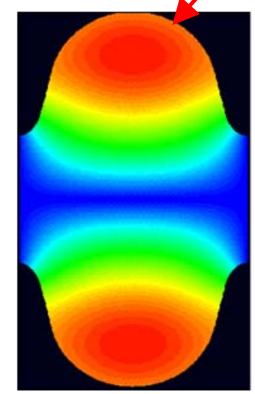
$$\frac{E_{peak}}{E_{acc}}$$




*Contour plot of /E/*

**$B_{peak}$ on the surface**

$$\frac{B_{peak}}{E_{acc}}$$



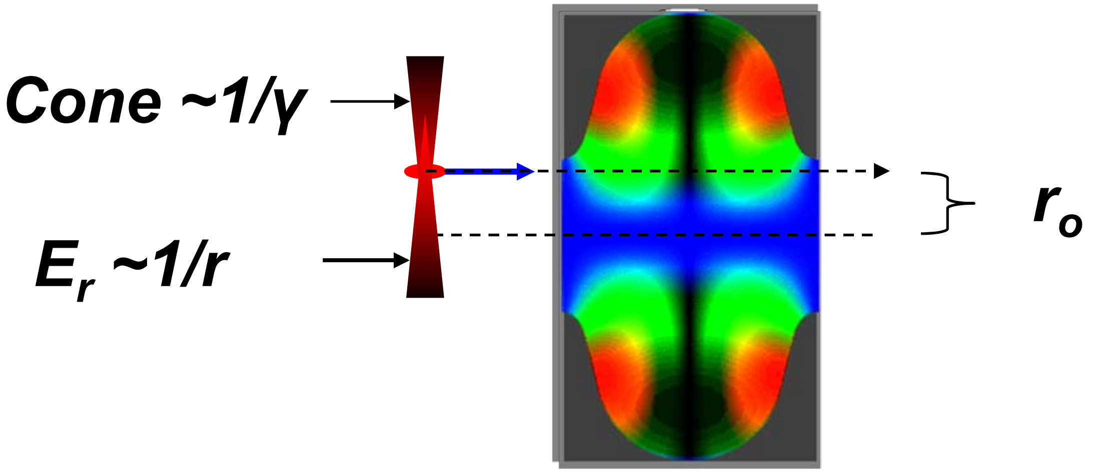
*Contour plot of /B/*

**Smaller value is preferred against electron emission phenomenon.**

**Ratio shows the limit in $E_{acc}$ due to the break-down of superconductivity (Nb ~2000 Oe ).**

# *Longitudinal and Transverse Loss Factors*

When ultra relativistic point charge **q** passes **empty cavity,**



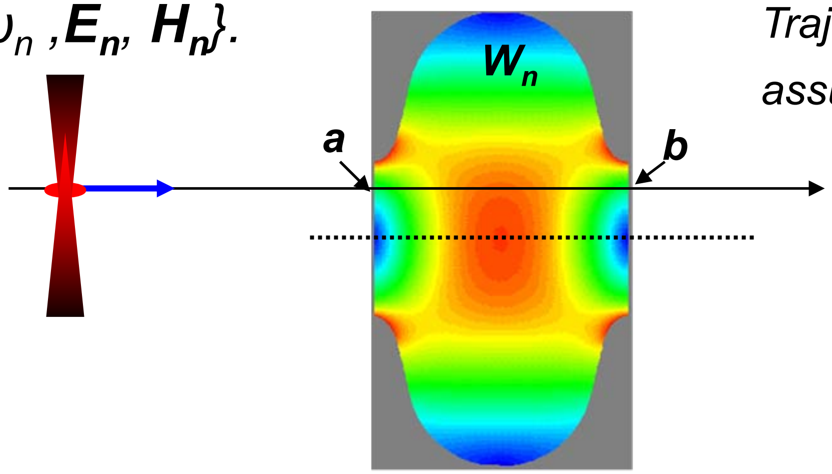

HOM modes are excited in the cavity.
If those are damped fast enough, the following beam feel them and can be kicked.
Thus the beam can be degradated.

# $(R/Q)_n$ of n-th HOM

***HOM nth $(R/Q)_n$, a "measure" of the energy exchange between point charge and mode n .***

*mode n : $\{\omega_n, \mathbf{E}_n, \mathbf{H}_n\}$.*

*Trajectory of the point charge q, assumed here to be a straight line.*




$$V_n = \sqrt{\left( \int_{z_a}^{z_b} E_{n,z} \sin\left(\frac{\omega_n}{\beta c}(z - z_a)\right) dz \right)^2 + \left( \int_{z_a}^{z_b} E_{n,z} \cos\left(\frac{\omega_n}{\beta c}(z - z_a)\right) dz \right)^2}$$

$$(R/Q)_n \equiv \frac{V_n^2}{\omega_n W_n}$$

# *Longitudinal and Transverse Loss Factors*

**J.Sekutwitz's Slide**

*The amount of energy lost by charge q to the cavity is:*

$\Delta U_q = k_{\|} \cdot q^2$ ***for monopole modes (max. on axis)***

$\Delta U_q = k_{\perp} \cdot q^2$ ***for non monopole modes (off axis)***

*where* $\mathbf{k}_{\|}$ *and* $\mathbf{k}_{\perp}(\mathbf{r})$ *are loss factors for the monopole and transverse modes respectively.*

*The induced* ***E-H field (wake)*** *is a superposition of* ***<u>cavity eigenmodes</u>*** ***(monopoles and others)*** *having the* $\mathbf{E}_n(\mathbf{r},\boldsymbol{\varphi},\mathbf{z})$ *field* ***<u>along the trajectory</u>.***

***For individual mode n and point-like charge:***

$$k^{\mathbf{p}}_{\|,n} = \frac{\omega_n \cdot (R/Q)_n}{4}$$

***Similar for other loss factors…….***

These harmful HOM power should be take out from the cavity.

Dr. Noguchi will make a lecture about the HOM coupler design.

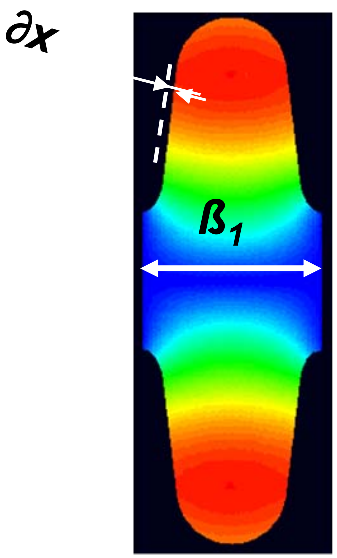
# Field Flatness Factor $a_{ff}$

J.Sekutwitz's Slide

*Field flatness factor for elliptical cavities with arbitrary ß=v/c*

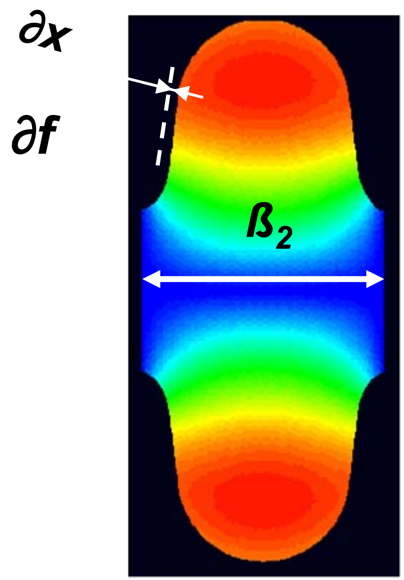
$$\frac{\Delta A}{A} = a_{ff} \cdot \frac{\Delta f}{f} \qquad a_{ff} = \frac{N^2}{k_{cc} \cdot ß}$$

***This is an empirical correction, based on intuition.***




***The same error ∂x causes bigger ∂f when a cell is thinner***

$$\frac{\partial f_1}{\partial f_2} = \frac{\partial x \cdot ß_2}{\partial x \cdot ß_1}$$

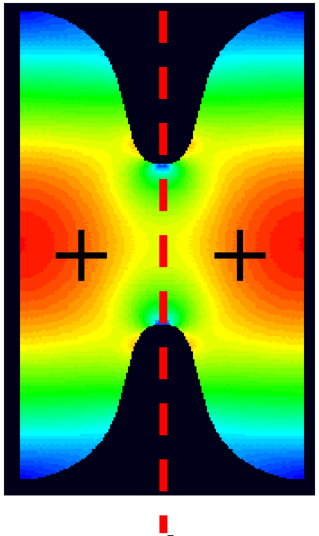



***Cells which geometric ß <1 are more sensitive to shape errors***

# Cell to cell coupling $k_{CC}$

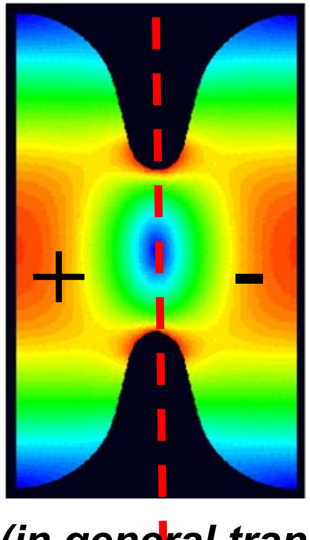
*The last parameter, relevant for multi-cell accelerating structures, is the coupling* $\boldsymbol{k_{cc}}$ *between cells for the accelerating mode pass-band (Fundamental Mode pass-band).*

$\boldsymbol{\omega_o}$




***no*** $\boldsymbol{E_r}$ ***(in general transverse E field) component at the symmetry plane***

$\boldsymbol{\omega_\pi}$




***no*** $\boldsymbol{H_\varphi}$ ***(in general transverse H field) component at the symmetry plane***

***The normalized difference between these frequencies is a measure of the Poynting vector (energy flow via the coupling region)***

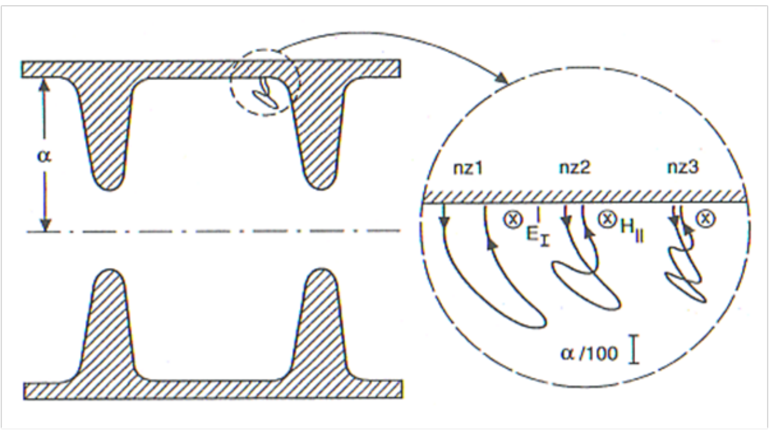
$$k_{cc} = \frac{\omega_\pi - \omega_0}{\dfrac{\omega_\pi + \omega_0}{2}}$$

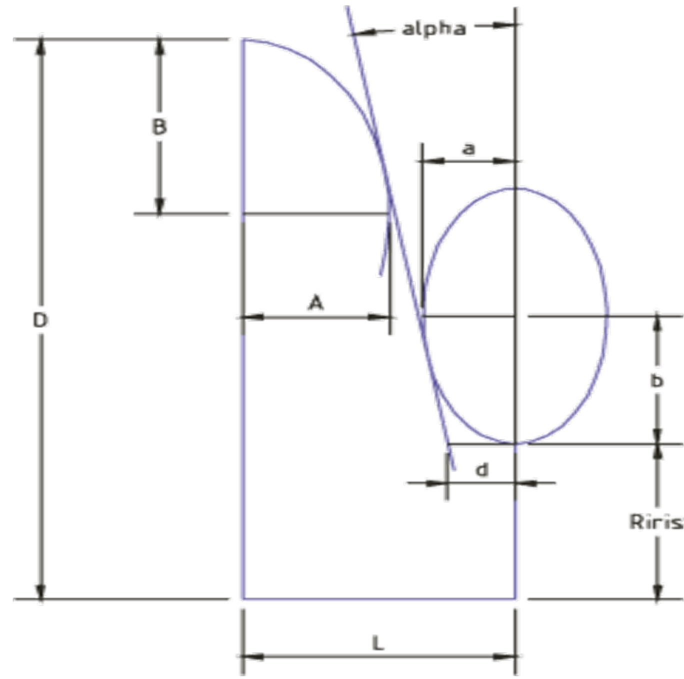
# Spherical/Elliptical Shape in SRF cavity design

Design of SRF cavity is usually done with spherical or Elliptical shapes.

**Choose spherical shape to reduce multipacting**

**Multipacting**




*R. Parodi* ***(1979)*** *presented first* ***spherical*** *C-band cavity with much less multipacting barrier than other cavities at that time.*

*P. Kneisel* ***(early 80's)*** *proposed for the DESY experiment the* ***elliptical shape*** *of 1 GHz cavity preserving good performance of the spherical one and stiffer mechanically.*



The RF cavity design tool will be given by Dr. U. van Rienen In this tutorial.

# Optimization of Cell Shape

*We begin with inner cells design because these cells "dominate" parameters of a multi-cell superconducting accelerating structure.*

**J.Sekutwitz's Slide**

***RF parameters summary:***

***FM :*** $(R/Q)$, $G$, $E_{peak}/E_{acc}$, $B_{peak}/E_{acc}$, $k_{cc}$

***HOM :*** $k_{\perp}$, $k_{\parallel}$.

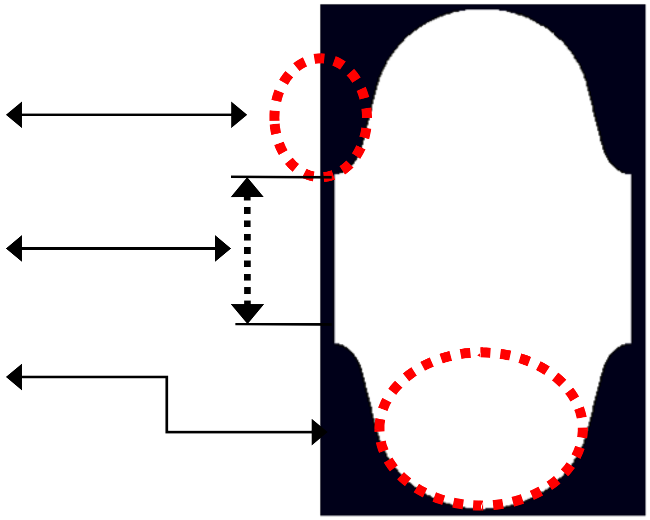
***There are 7 parameters we want to optimize for a inner cell***

***Geometry :***

***iris ellipsis : half-axis*** $h_r$, $h_z$

***iris radius :*** $r_i$

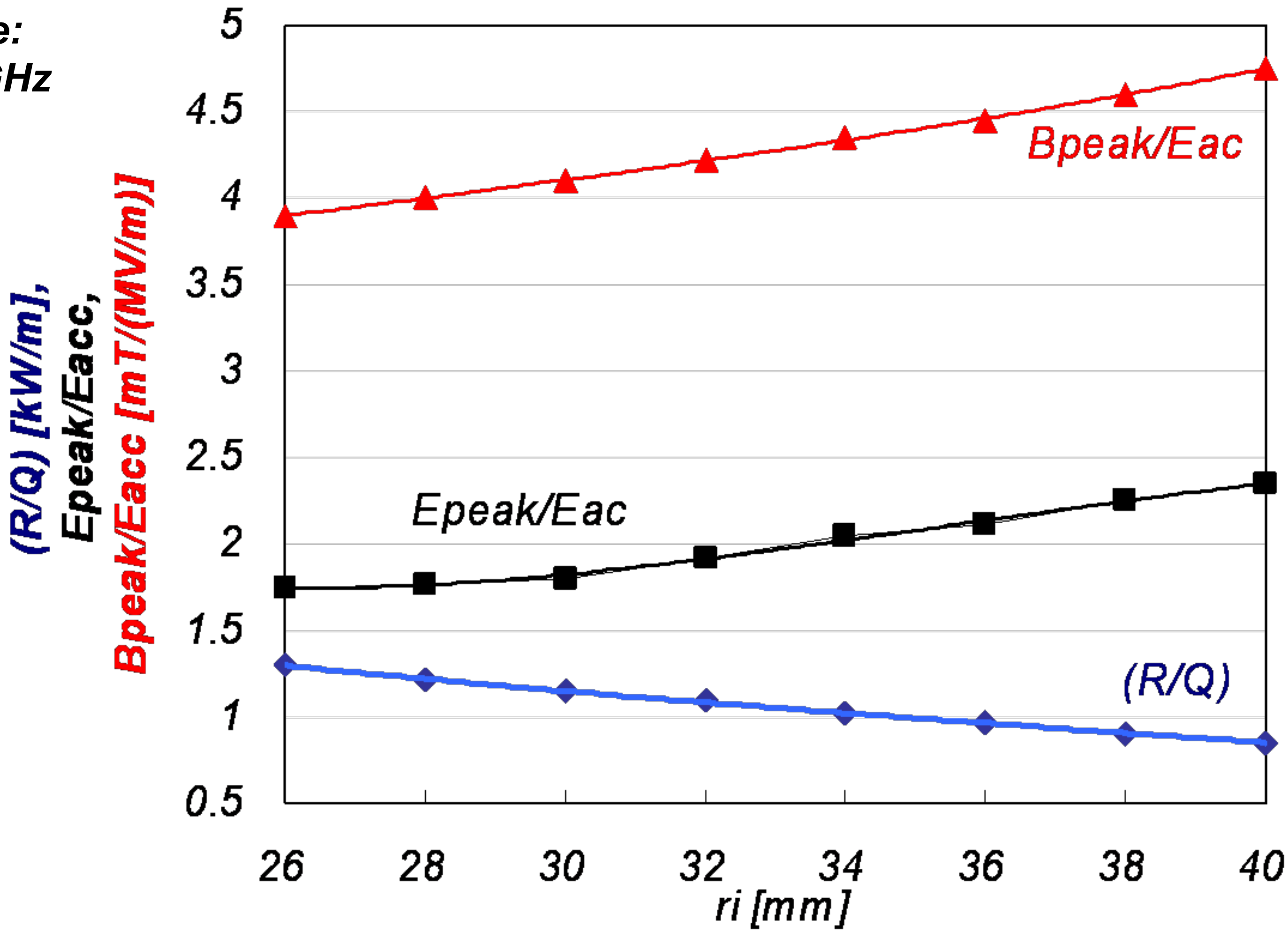
***equator ellipsis : half-axis*** $h_r$, $h_z$



***There is some kind of conflict <u>7 parameters</u> and only <u>5 variables</u> to "tune"***

# Effect of Cavity Aperture on RF Parameters

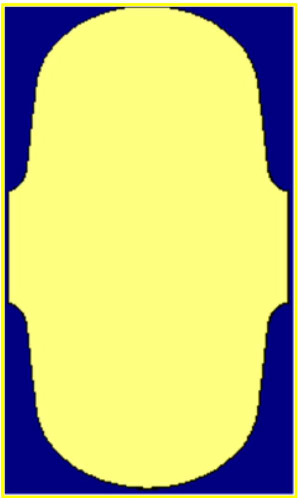
***Example:***
***f = 1.5 GHz***

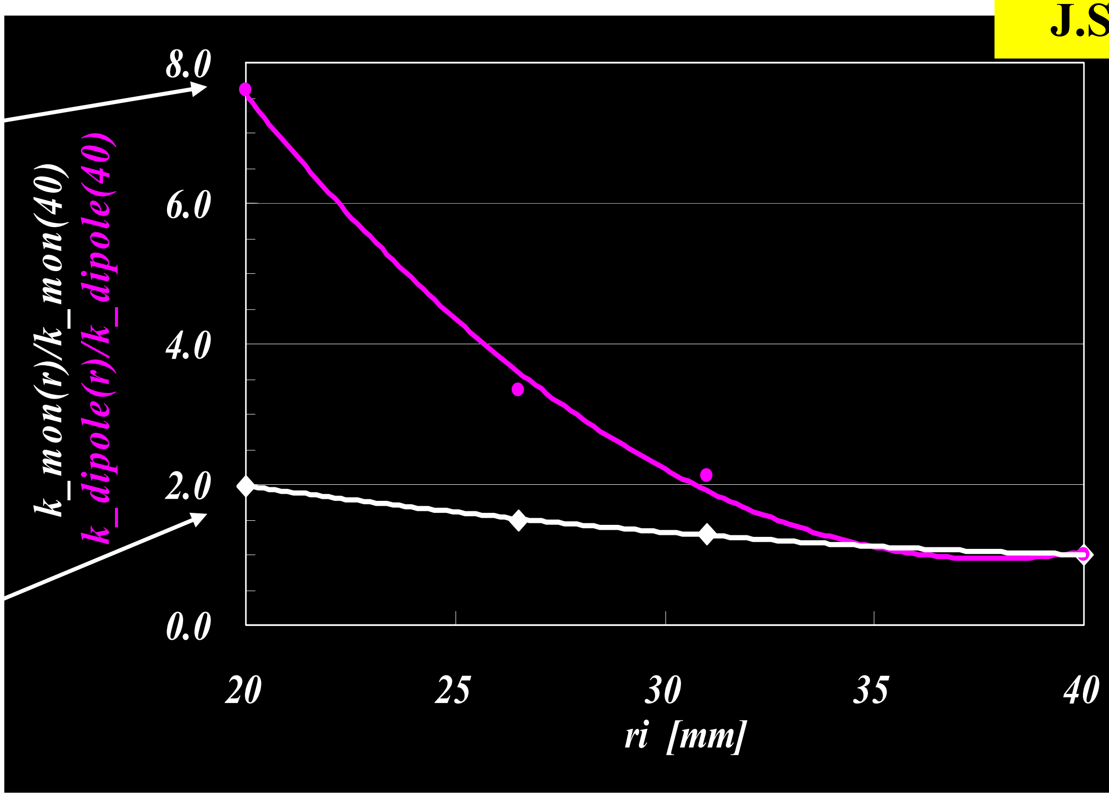


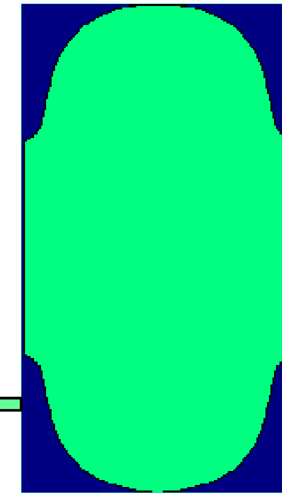

***A. Mosnier, E. Haebel, SRF Workshop 1991***

# Aperture Effects on $\kappa_{//}$ and $\kappa_{\perp}$)

**J.Sekutwitz's Slide**



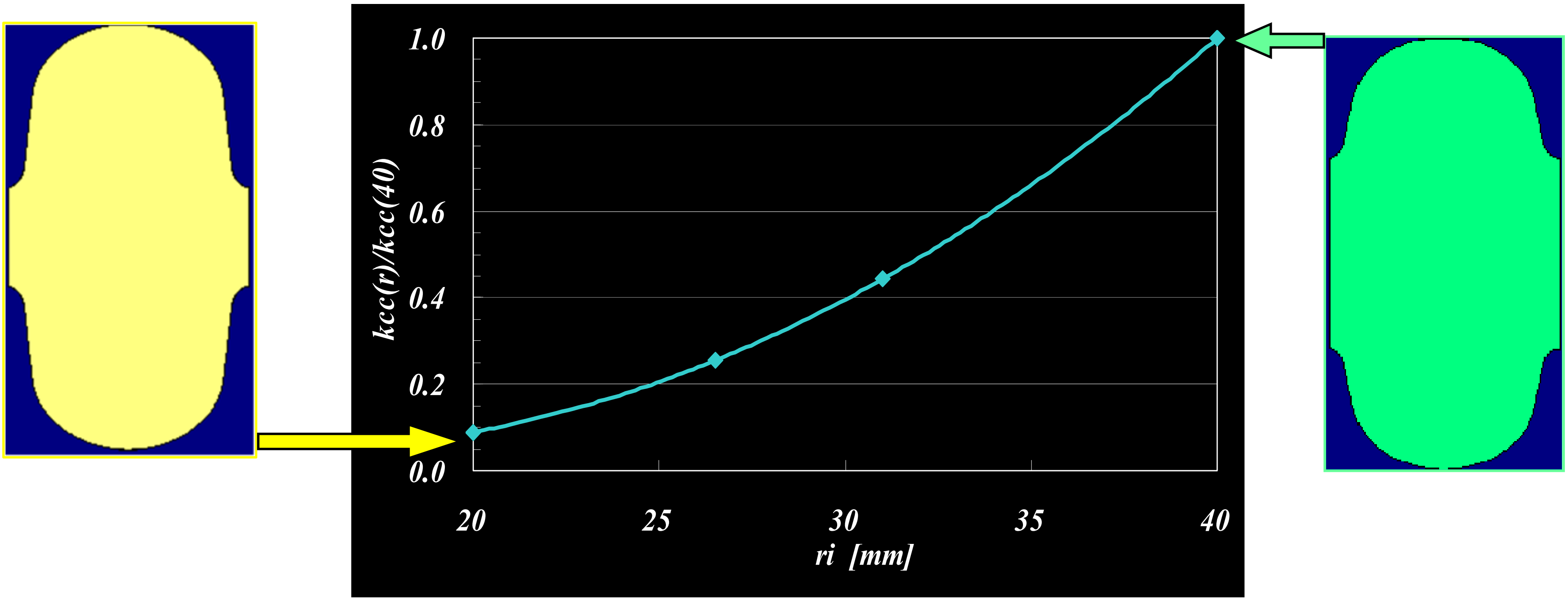


| | |
|---|---|
| *(R/Q) = 152 Ω* | *(R/Q) = 86 Ω* |
| $B_{peak} / E_{acc}$ = 3.5 mT/(MV/m) | $B_{peak} / E_{acc}$ = 4.6 mT/(MV/m) |
| $E_{peak} / E_{acc}$ = 1.9 | $E_{peak} / E_{acc}$ = 3.2 |

# Aperture Effect on Cell to Cell Coupling ($\kappa_{CC}$)

**J.Sekutwitz's Slide**




*(R/Q) = 152 Ω*

*$B_{peak}$ / $E_{acc}$ = 3.5 mT/(MV/m)*

*$E_{peak}$ / $E_{acc}$ = 1.9*

*(R/Q) = 86 Ω*

*$B_{peak}$ / $E_{acc}$ = 4.6 mT/(MV/m)*

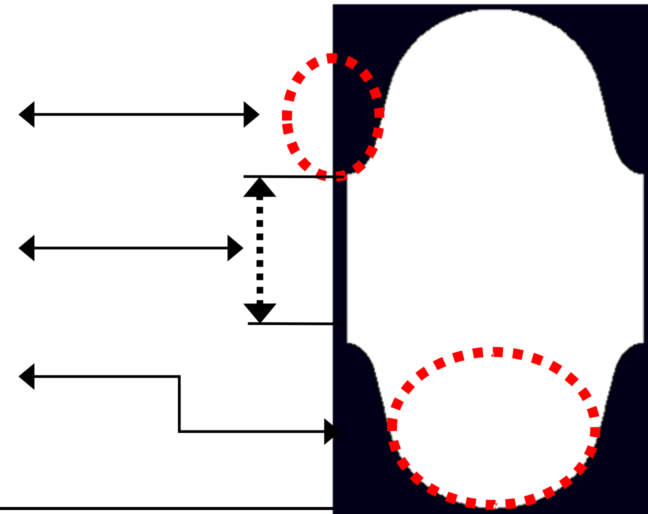
*$E_{peak}$ / $E_{acc}$ = 3.2*

# General Trends of Cavity Optimization on RF Geometrical Parameters

*iris ellipsis* : *half-axis* $h_r, h_z$

*iris radius* : $r_i$

*equator ellipsis* : *half-axis* $h_r, h_z$



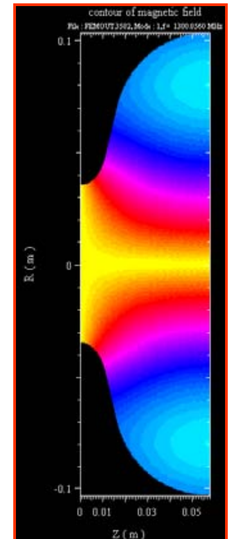
| *Criteria* | *RF-parameter* | *Improves when* | *Cavity examples* |
|---|---|---|---|
| *Operation at high gradient* | $E_{peak}/E_{acc}$ $B_{peak}/E_{acc}$ ↓ | $r_i$ ↓ *Iris, Equator shape* | *TESLA, HG CEBAF-12 GeV* |
| *Low cryogenic losses* | $(R/Q)\cdot\Gamma$ ↑ | $r_i$ *Equator shape* ↓ | *LL CEBAF-12 GeV* *LL- ILC cavity* |
| *High* $I_{beam}$ ↔ *Low HOM impedance* | $k_{\perp}, k_{\parallel}$ ↓ | $r_i$ ↑ | *B-Factory* *RHIC cooling* |

# Choice of the RF Frequency

*What about accelerating mode frequency of a superconducting cavity?*

**J.Sekutwitz's Slide**

**x 2 =**



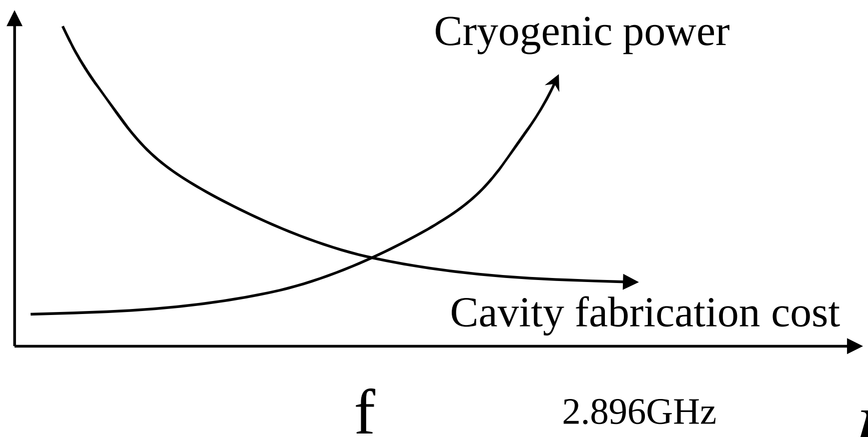

| | | |
|---|---|---|
| $f_\pi$ | *[MHz]* | *2600* |
| *R/Q* | *[Ω]* | *57* |
| *r/q=(R/Q)/l* | *[Ω/m]* | *2000* |
| *G* | *[Ω]* | *271* |

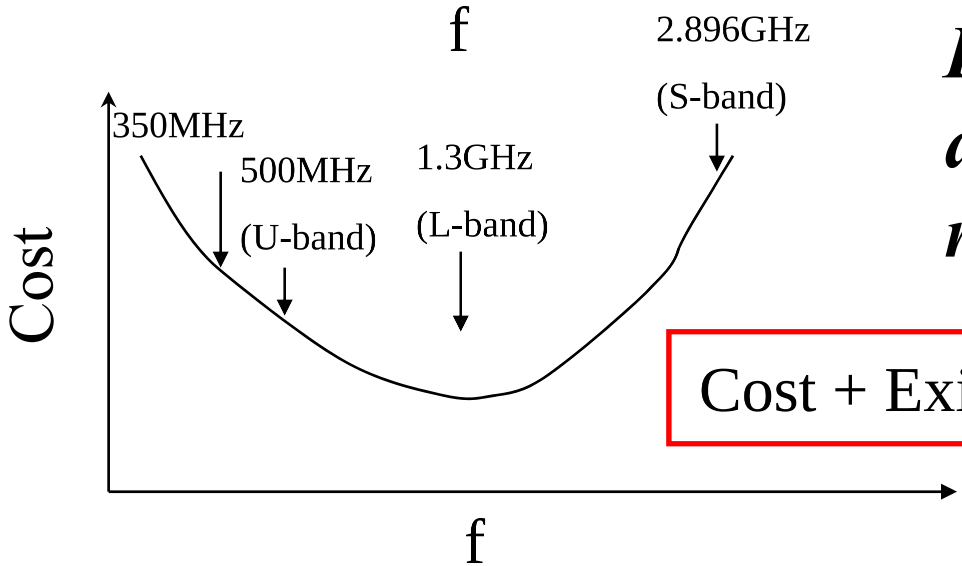
| | | |
|---|---|---|
| $f_\pi$ | *[MHz]* | *1300* |
| *R/Q* | *[Ω]* | *57* |
| *r/q=(R/Q)/l* | *[Ω/m]* | *1000* |
| *G* | *[Ω]* | *271* |

***r/q=(R/Q)/l ~ f***

# Frequency Choice of the SRF Cavity

$$P_{loss} = \frac{1}{2} R_s \int_S H_S dS \propto E_{acc}^2, \qquad R_S(BCS) = \frac{A}{T} \cdot f^2 \cdot \exp(-\frac{\Delta}{k_B T})$$




***L-band cavity is preferred and it is operated at 2K to reduce the cryogenic power.***

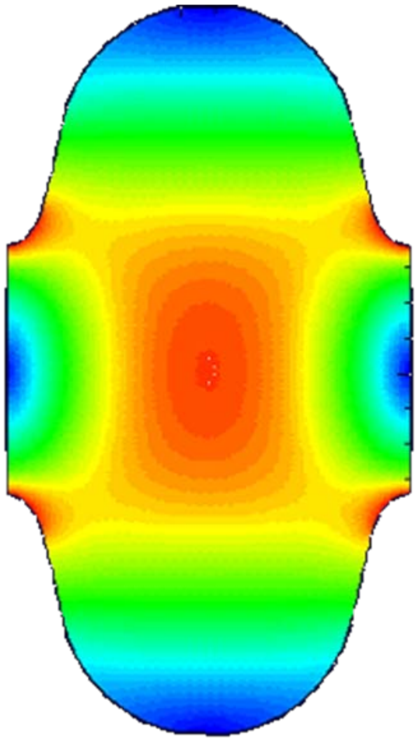
Cost + Existing RF source

# TESLA Cavity Inner Cell Shape

***The inner cell geometry*** *was optimize with respect to:* ***low*** $E_{peak}/E_{acc}$ *and coupling* $k_{cc}$.

*At that time (1992) the field emission phenomenon and field flatness were of concern, no one was thinking about reaching the magnetic limit.*

| | | |
|---|---|---|
| $f_{\pi}$ | [MHz] | 1300.0 |
| $r_{iris}$ | [mm] | 35 |
| $k_{cc}$ | [%] | 1.9 |
| $E_{peak}/E_{acc}$ | - | 1.98 |
| $B_{peak}/E_{acc}$ | [mT/(MV/m)] | 4.15 |
| R/Q | [Ω] | 113.8 |
| G | [Ω] | 271 |
| R/Q*G | [Ω*Ω] | 30840 |



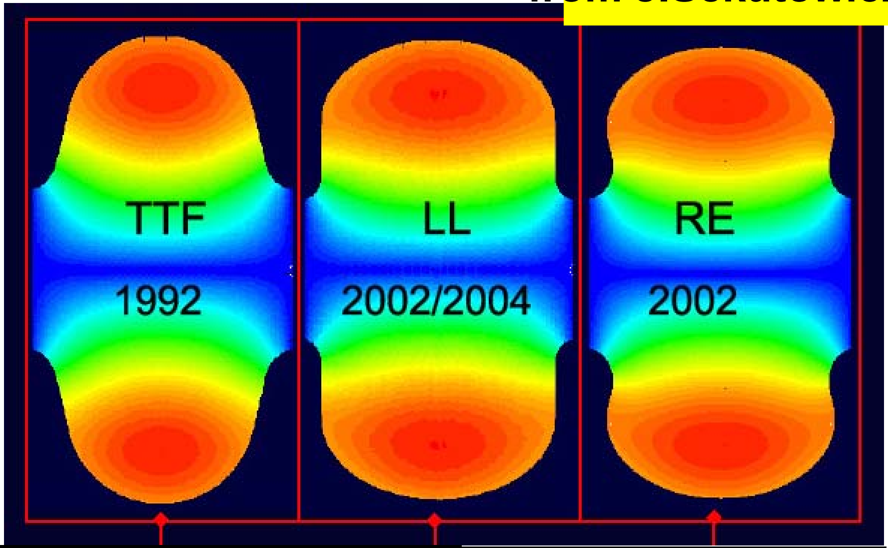
*Inner cell; Contour of E field*

# High Gradient Shapes

## Cavity shape designs with low Hp/Eacc

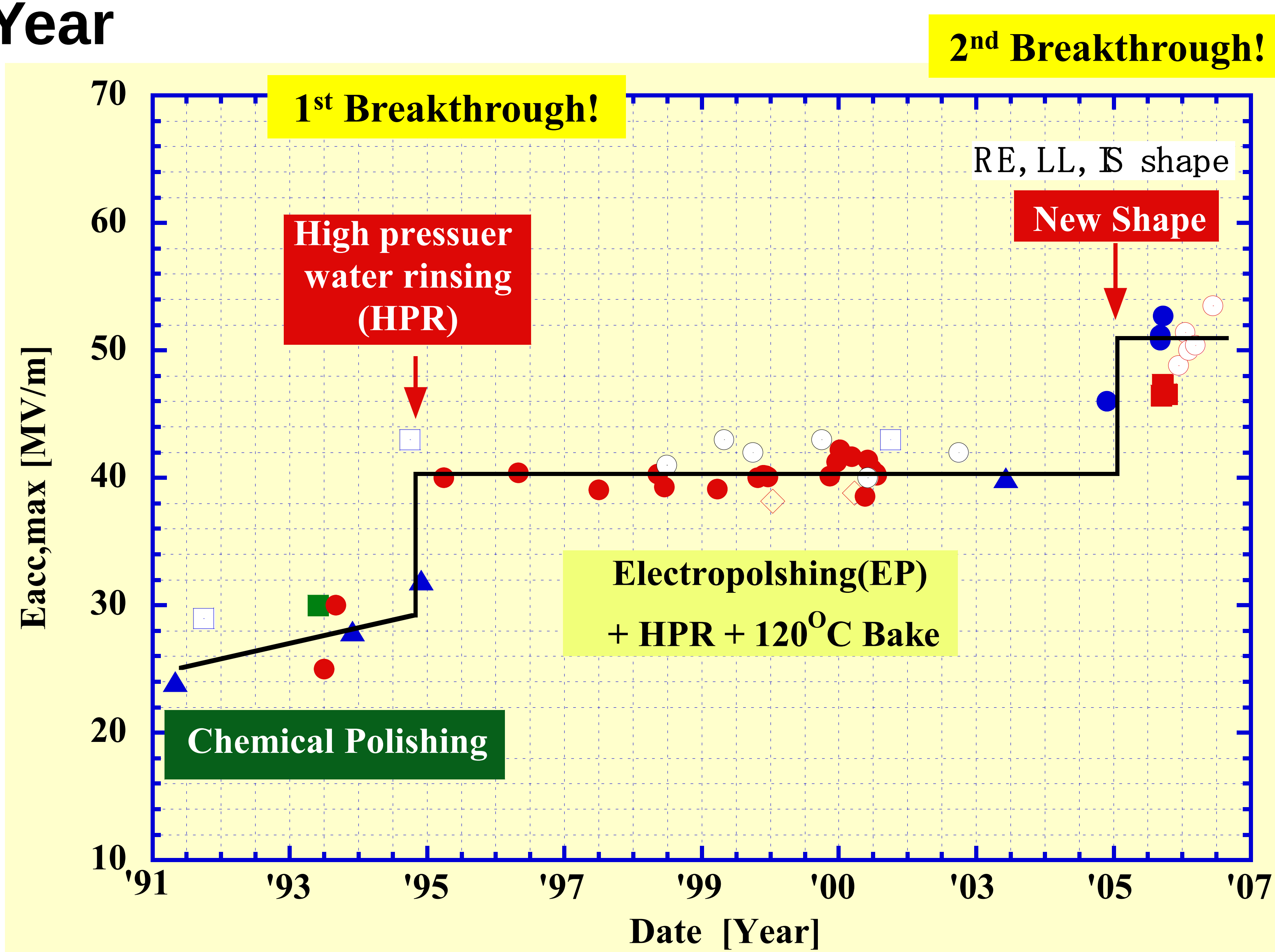
from J.Sekutowicz lecture Not

TTF: TESLA shape
Reentrant (RE): Cornell Univ.
Low Loss(LL): JLAB/DESY
Ichiro-Single (IS) : KEK




| | TESLA | LL | RE | IS |
|---|---|---|---|---|
| Diameter [mm] | 70 | 60 | 66 | 61 |
| Ep/Eacc | 2.0 | 2.36 | 2.21 | 2.02 |
| Hp/Eacc [Oe/MV/m] | 42.6 | 36.1 | 37.6 | 35.6 |
| R/Q [W] | 113.8 | 133.7 | 126.8 | 138 |
| G[W] | 271 | 284 | 277 | 285 |
| Eacc max | 41.1 | 48.5 | 46.5 | 49.2 |

# Eacc vs. Year


2nd Breakthrough!
1st Breakthrough!
RE, LL, IS shape
New Shape
High pressuer water rinsing (HPR)
Electropolshing(EP)
+ HPR + 120OC Bake
Chemical Polishing
Eacc,max [MV/m]
70
60
50
40
30
20
10
'91
'93
'95
'97
'99
'00
'03
'05
'07
Date [Year]

# 4. Criteria for Multi-cell Structures

*Single-cell is attractive from the RF-point of view:*

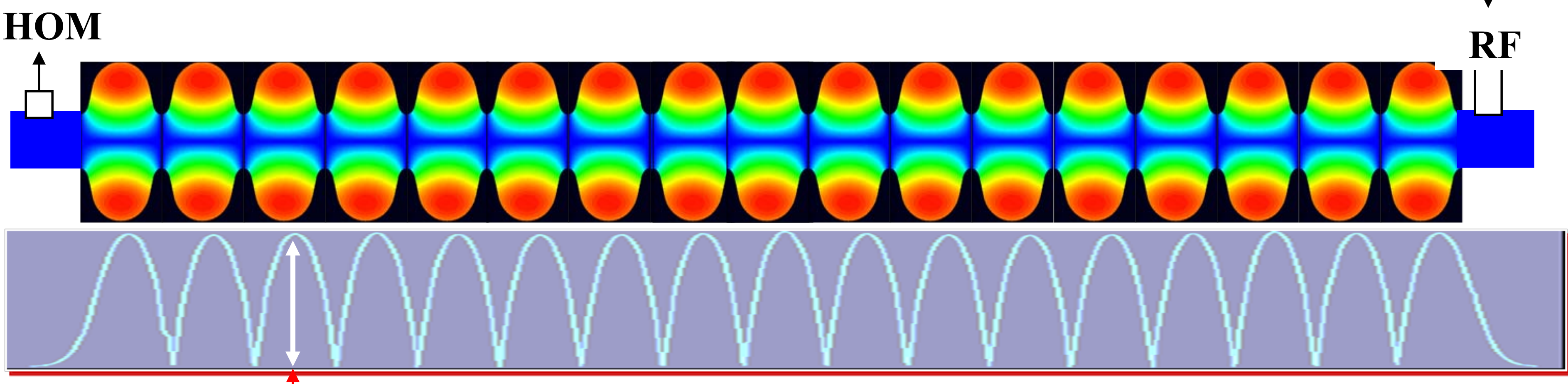
- *Easier to manage HOM damping*
- *No field flatness problem.*
- *Input coupler transfers less power*
- *Easy for cleaning and preparation*
- *But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.*

*A multi-cell structure is less expensive and offers higher real-estate gradient but:*

- *Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells*
- *Other problems arise: HOM trapping…*

**How to decide the number of cells ?**

# Parameters considered with N




$$\frac{\Delta A_i}{A_i} = a_{ff} \frac{\Delta f_i}{f_i} = \frac{N^2}{k_{cc}} \cdot \frac{\Delta f_i}{f_i}$$

**Beam pipe has no acceleration beam.**
**BP reduce the efficiency.**
**Multi-cell is more efficient.**

**Field flatness factor :** $$a_{ff} = \frac{N^2}{k_{cc}}$$

**HOM trapped mode :** More serious within creasing N

**Input handling power :** $P_{INPUT} \propto$ N

**Handling :** Easy to bend, Difficult to preparation …..

# Effect of N on Field Flatness Sensitivity Factor

**J.Sekutwitz's Slide**

- Field flatness vs. N

| | Original Cornell N = 5 | High Gradient N =7 | Low Loss N =7 | TESLA N=9 | SNS ß=0.61 N=6 | SNS ß=0.81 N=6 | RIA ß=0.47 N=6 | RHIC N=5 |
|---|---|---|---|---|---|---|---|---|
| year | 1982 | 2001 | 2002 | 1992 | 2000 | 2000 | 2003 | 2003 |
| $a_{ff}$ | 1489 | 2592 | 3288 | 4091 | 3883 | 2924 | 5040 | 850 |

*Many years of experience with: heat treatment, chemical treatment, handling and assembly allows one to preserve tuning of cavities, even for those with bigger N and weaker $k_{cc}$*

***For the TESLA cavities: field flatness is better than 95 %***

# HOM trapping vs. N

**J.Sekutwitz's Slide**

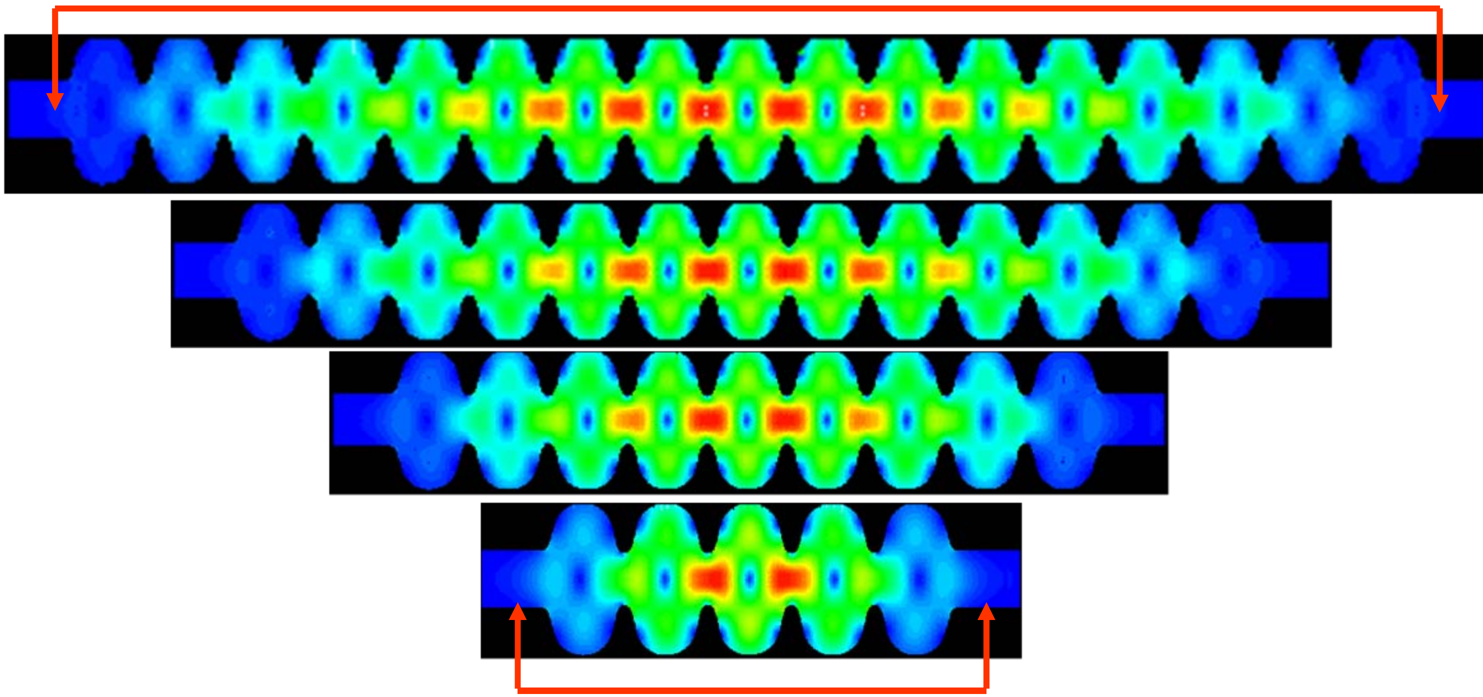
**No fields at HOM couplers positions, which are always placed at end beam tubes**

*N = 17*

*N = 13*

*N = 9*

*N = 5*



**e-m fields at HOM couplers positions**

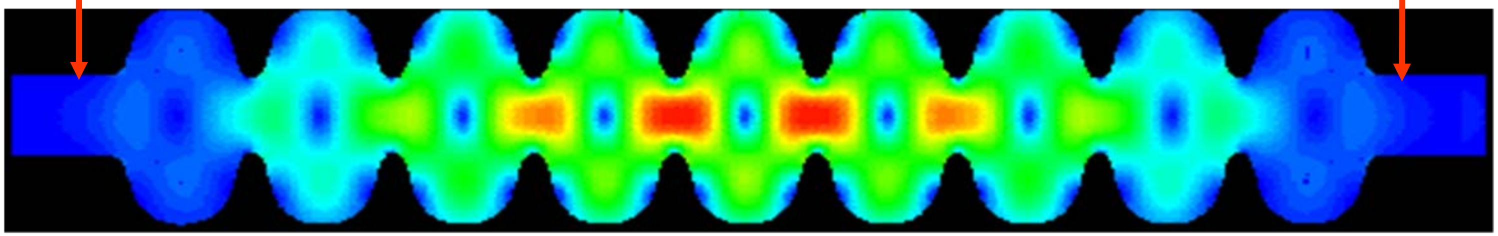
Cure for the trapped mode: Make the bore radius larger.
Break mirror symmetry.

# HOM Issue with Multi-Cell Structure

*HOM couplers limit RF-performance of sc cavities when they are placed on cells*

**J.Sekutwitz's Slide**

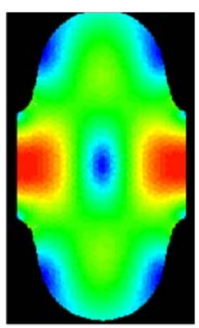
*no E-H fields at HOM couplers positions, which are always placed at end beam tubes*



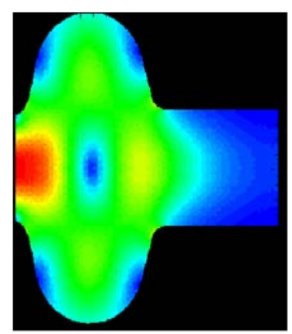
*The HOM trapping mechanism is similar to the FM field profile unflatness mechanism:*

- *weak coupling HOM cell-to-cell,* $\boldsymbol{k}_{cc,HOM}$
- *difference in HOM frequency of end-cell and inner-cell*

*f = 2385 MHz*



*That is why they hardly resonate together*

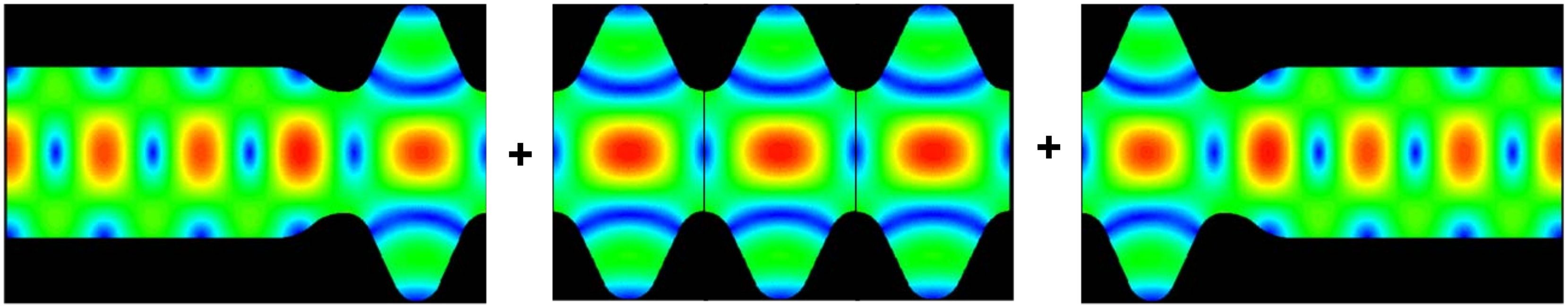


*f = 2415 MHz*

# Adjustment of End-Cells

J.Sekutwitz's Slide

*The geometry of end-cells differs from the geometry of inner cells due to the attached beam tubes*




*Their function is multi-folded and their geometry must fulfill three requirements:*

- *field flatness and frequency of the accelerating mode*
- *field strength of the accelerating mode at FPC location enabling operation with matched Qext*
- *fields strength of dangerous HOMs ensuring their required damping by means of HOM couplers or/and beam line absorbers.*

*All three make design of the end-cells more difficult than inner cells.*

# Capable Input Power on N

- *Power capability of fundamental power couplers vs. N*

***When $I_{beam}$ and $E_{acc}$ are specified and a superconducting multi-cell structure does not operate in the energy recovery mode:***

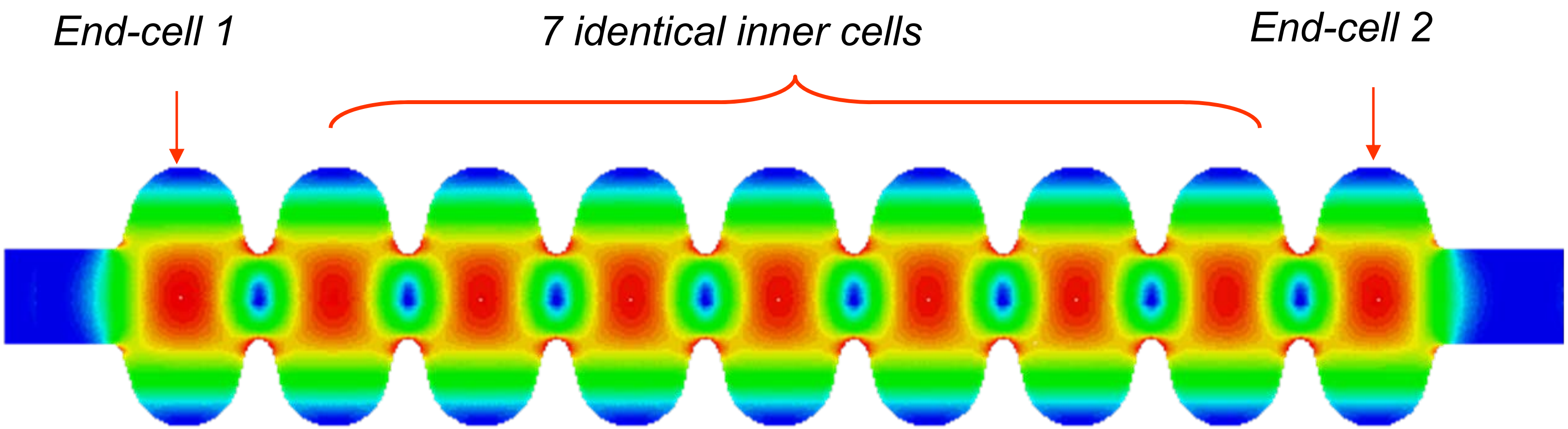
$$P_{INPUT} \sim N$$

**Coupler handling power limits the small N in the intensity frontier machine like KEK B(N=1).**

KEKB: Acceleration beam current > 1 A.

# TESLA/ILC Baseline Cavity

*The cavity was designed in 1992 (A. Mosnier, D. Proch and J.S. ).*




*TTF 9-cells; Contour of E field*

| | | |
|---|---|---|
| $f_{\pi}$ | [MHz] | 1300.00 |
| $f_{\pi\text{-}1}$ | [MHz] | 1299.24 |
| $R/Q$ | [Ω] | 1012 |
| $G$ | [Ω] | 271 |
| *Active length* | [mm] | 1038 |

# Overview of Cavities

**J.Sekutwitz's Slide**

***Examples of Inner cells***

| | | CEBAF Original Cornell ß=1 | new<br>CEBAF -12 High Gradient ß=1 | new<br>CEBAF -12 Low Loss ß=1 | TESLA ß=1 | SNS ß=0.61 | SNS ß=0.81 | new<br>RIA ß=0.47 | new<br>RHIC Cooler ß=1 |
|---|---|---|---|---|---|---|---|---|---|
| $f_o$ | [MHz] | 1448.3 | 1468.9 | 1475.1 | 1278.0 | 792.8 | 792.8 | 793.0 | 683.0 |
| $f_\pi$ | [MHz] | 1497.0 | 1497.0 | 1497.0 | 1300.0 | 805.0 | 805.0 | 805.0 | 703.7 |
| $k_{cc}$ | [%] | 3.29 | 1.89 | 1.49 | 1.9 | 1.52 | 1.52 | 1.52 | 2.94 |
| $E_{peak}/E_{acc}$ | - | 2.56 | 1.96 | 2.17 | 1.98 | 2.66 | 2.14 | 3.28 | 1.98 |
| $B_{peak}/E_{acc}$ | [mT/(MV/m)] | 4.56 | 4.15 | 3.74 | 4.15 | 5.44 | 4.58 | 6.51 | 5.78 |
| R/Q | [Ω] | 96.5 | 112 | 128.8 | 113.8 | 49.2 | 83.8 | 28.5 | 80.2 |
| G | [Ω] | 273.8 | 266 | 280 | 271 | 176 | 226 | 136 | 225 |
| R/Q*G | [Ω*Ω] | 26421 | 29792 | 36064 | 30840 | 8659 | 18939 | 3876 | 18045 |
| $k_\perp$ ($\sigma_z$=1mm) | [V/pC/cm²] | 0.22 | 0.32 | 0.53 | 0.23 | 0.13 | 0.11 | 0.15 | 0.02 |
| $k_\parallel$ ($\sigma_z$=1mm) | [V/pC] | 1.36 | 1.53 | 1.71 | 1.46 | 1.25 | 1.27 | 1.19 | 0.85 |

**β vs RF parameters**

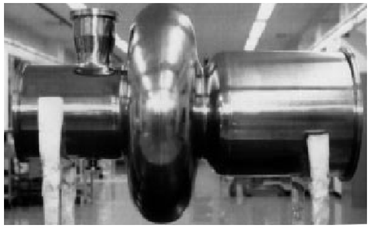
β smaller →

- Ep/Eacc larger
- Hp/Eacc larger
- (R/Q) smaller

# 5. Example of SRF High-beta Cavities

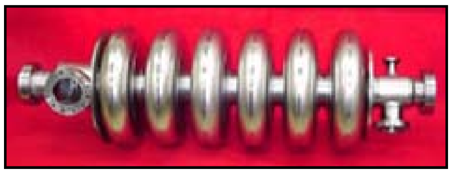
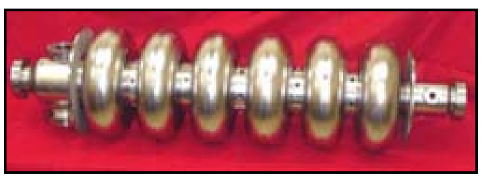
**J.Sekutwitz's Slide**

*Cavities operating with highest $I_{beam}$ or $E_{acc}$*

| *Type /No. of cavities* | | | *$P_{beam}$/cavity [kW]* | *$P_{HOM}$ /cavity [kW]* |
|---|---|---|---|---|
| **KEK-B 0.5 GHz**  *x 8* | *Single-cell with max $I_{beam}$* | *$I_{beam}$= 1.34 A* *1389 bunches* *cw* | *350* | *16* |
| **HERA 0.5 GHz**  *x 16* | *Multi-cell with max $I_{beam}$* | *$I_{beam} \leq 40$ mA* *180 bunches* *cw* | *60* | *0.13* |
| **TTF-I , 1.3 GHz**  *x 1* | *Multi-cell with max $E_{acc}$* | *$E_{acc}$= 35 MV/m* *1.3ms/pulse 1Hz PRF* | *~100* *Almost no beam loading* | *0* |

J.Sekutwitz's Slide

## Cavities which will operate with high $I_{beam}$ in the near future

| *Type /No. cavities* | | | *$P_{beam}$/cavity [kW]* | *$P_{HOM}$ /cavity [kW]* |
|---|---|---|---|---|
| **SNS ß= 0.61, 0.805 GHz**  *x 33* | *Multi-cell with max $I_{beam}$* | *$I_{beam}$=38 (59 ) mA 1.3ms/pulse DF = 6 %* | *240 (366)* | *0.06 peak* |
| **SNS ß= 0.805 GHz**  *x 48* | | | *482* | *0.06 peak* |
| **TTF-II ep , 1.3 GHz**  *x 8* | *Multi-cell with max $E_{acc}$* | *$E_{acc}$= 35 MV/m 1.3ms/pulse 10Hz PRF* | *146* | *< 0.02>* |