Strain hardening during constrained deformation of metal foams – effect of shear displacement

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The crush bands that form during plastic deformation of closed-cell metal foams are often inclined at 11–20° to the loading axis, allowing for shear displacement of one part of the foam with respect to the other. Such displacement is prevented by the presence of a lateral constraint. This was analysed in this study, which shows that resistance against shear by the constraint leads to the strain hardening effect in the foam that has been reported in a recent experimental study.

Keywords: metal foam; constrained deformation; shear; band; strain hardening

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Closed-cell metal foams have received considerable attention as they offer the advantage of high specific energy absorption. This is because of their ability to undergo large plastic strains (typically ~60%, or even more) at a near-constant stress level under quasi-static compression. The large plateau in the plastic part of the stress-strain curve is due to the collective cell collapse in bands and propagation of cell crushing from one band to another. Normally, strain hardening in metal foams is insignificant. However, when the foam specimen is subjected to a lateral constraint, stresses required for plastic deformation increase with strain, implying an inducement of strain hardening by the constraint [1]. Such strain hardening has important implications especially in the context of fatigue [2].

Karthikeyan et al. [3] identified multi-axial states of stress and frictional resistance between the deforming foam and the rigid constraint walls as the two main sources for the observed strain hardening. They analysed this by considering the cell collapse bands to be perpendicular to the loading direction. This, in turn, automatically precludes the possibility of any bulk shear displacement during deformation. However, experimental evidence suggests that these bands may not be necessarily perpendicular to the loading axis. Often, the normal to the crush band planes is inclined with respect to the loading axis, both in the constrained [1,4] as well as unconstrained [5] conditions. In the former case, the angle of inclination is small, as shown by metallography and X-ray tomography of foams subjected to constrained deformation [1,4]. This inclination converts the compression loading into a shear along the inclined crush band. When there is no constraint, shear is allowed to take place. The presence of constraint can modify the stress state by preventing shear displacement. We investigate this possibility and examine the role of shear band inclination on strain hardening behaviour of metal foams in this paper.

A closed-cell *ALPORAS*® aluminium alloy foam supplied by Shinko Wire (Japan) was used in this study. Processing details and relevant properties of this foam can be found in Ref. [6]. Four samples of 50 × 50 mm^2 cross section and 100 mm height were electro-discharge machined from a single large plate of *ALPORAS* foam. The thickness of the plate coincides with the loading direction. Two samples were tested with quasi-static compression loading, while other two with compression-compression fatigue loading. A die-steel sleeve of 50.8 × 50.8 mm^2 inner cross section and 118 mm depth was used as lateral constraint during deformation. The inner area of the sleeve was chosen in such a way so that all samples could be fitted into the sleeve easily. Samples were fixed into the sleeve with the help of screws (which enabled easy removal of the deformed samples). After that a solid aluminium block of 50 × 50 mm^2 was placed on top of the foam sample. This entire setup was placed between parallel rigid platens of the universal testing machine and tests were performed.

Compression was performed at a rate of 0.1 mm/s. For fatigue tests, the load ratio (defined as the ratio of the minimum to the maximum loads of the sinusoidal fatigue cycle), was maintained at 0.1. Each sample was tested at a frequency of 10 Hz up to 10^5 cycles. Both the samples were tested at a maximum stress to plastic collapse strength (σ_p, calculated according to Ref. [1]) ratio of 0.9. After subjecting different amount of strain, test samples were removed from the sleeve and were imaged using X-ray radiography to measure the accumulated strain, deformation inside the foam, crush (or shear) band formation and shear displacement. A microfocus X-ray source (100 kV voltage, 100 μA current and 5 μm spot size) and a flat panel detector (area 120 × 120 mm^2, pixel size 50 μm), both supplied by Hamamatsu (Japan), were used for imaging. After acquiring images, deformation was continued by re-mounting the sample into the sleeve. Radiographic images of the same sample in more deformed states were acquired after further deformation.

Relative density of all the four foams is similar with small deviation, 0.09 ± 0.003. Results for one sample from each test group are shown in Fig. 1. Due to the cone geometry of the X-ray beam, only one surface can be aligned perfectly with the beam. Figs. 1a and 1c show that the undeformed foams have flat surface, whereas the same surface becomes uneven after plastic deformation, see Figs. 1b and 1d. Localized deformation took place and formed plastic collapse bands, which can be seen as darker regions in Fig. 1b and 1d. Although the gap between foam surface and constraint wall is small (maximum ~0.8 mm), it is sufficient for some shear displacement to be accommodated. Shear displacement leads to uneven foam surfaces, which in turn prevents the contact of the full surface of the foam with the constraint wall. As a result, the frictional resistance against the vertical movement of the foam is contributed only by the part of the foam touching the constraint wall. Shear displacement was also observed after all the interrupted deformation stages. Results are presented only for the highest applied strain, when it is more clearly visible. Results are consistent with the observation of the other two samples. Hence, shear displacement is common in both compression and fatigue loading even in the presence of constraint. In the following we demonstrate the implication of shear displacement and its prevention by the constraint.

Stress-strain response up to σ_p is the same both in constrained and unconstrained deformation [1]. After reaching σ_p, the stress-strain response of constrained sample deviates from that of the unconstrained one: the stress-strain curve of constrained sample exhibits a much more pronounced positive slope, i.e., strain hardening, than the unconstrained one. We assume that there is only one crush band in the foam, inclined at an angle α to the x-axis in the x–z plane and is parallel to the y–z face as shown schematically in Fig. 2a. Note that only one side wall of the constraint separated by a small gap is shown. Beyond σ_p, shear displacement takes place and the upper part of the foam shears along the inclination of the crush band (Fig. 2b) until it touches the constraint wall.

The orientation of the forces (or stresses) in relation to the coordinate system is indicated in the subscript. For example, F_{1-x} indicates force F_1 along x direction, whereas F_{1-xz} denotes force F_1 in the x–z plane. The components of the vertical load are indicated along the crush band in Fig. 2b. Let F_z be the force at the point when the vertical stress reaches σ_{p-z}. Shear displacement of the upper part of the foam is accompanied by a rotation of the crush band distorting its rectangular shape. This rotation is small due to the limited shear displacement in constrained deformation and hence is ignored here. The component of F_z along the band has to overcome the shear strength ($\tau_{shear-xz}$) of the foam that is already deformed. The resultant force F_{1-xz} along the inclination of the crush band is given by

$$F_{1-xz} = F_z \cdot \sin\alpha - \tau_{shear-xz} \cdot \frac{a^2}{\cos\alpha}, \qquad (1)$$

where a^2 is the cross section of the foam. The area of the inclined crush band is $a^2/\cos\alpha$. If shear displacement occurs then F_{1-xz} must be positive. Typically, the shear strength of a foam (undeformed) is about two-thirds of the plateau strength [6,7], i.e., $2\sigma_p/3$. In Eq. 1, $\tau_{shear-xz}$ is the shear strength of the deformed foam. Therefore, $\tau_{shear-xz}$ is likely to differ from the theoretical shear strength of the foam. The values for the crush band inclination and $\tau_{shear-xz}$ determine when shearing starts. The limiting values of α and $\tau_{shear-xz}$ can be calculated by setting $F_{1-xz} = 0$ in Eq. 1, which results in the following relationship

$$F_z \cdot \sin\alpha = \tau_{shear-xz} \cdot \frac{a^2}{\cos\alpha}. \qquad (2)$$

Ideally, in the plateau region of the stress–strain curve, F_z is given by $\sigma_{p-z} \cdot a^2$. This leads to a relation

$$\tau_{shear-xz} = \frac{1}{2} \cdot \sin 2\alpha \cdot \sigma_{p-z}. \qquad (3)$$

Eq. 3 gives the lowest value of $\tau_{shear-xz}$ for a particular α until which shear displacement is not possible. Note that Eq. 3 gives the shear strength (of deformed foam) as a fraction of compressive strength of the foam. For shearing to occur for a particular value of α, $\tau_{shear-xz}$ has to be less than $1/2 \cdot \sin 2\alpha \cdot \sigma_{p-z}$. For instance, according to Eq. 3 the necessary criterion for shearing at 15° angle is that $\tau_{shear-xz} < 0.25 \cdot \sigma_{p-z}$. In this way one can assume a value of $\tau_{shear-xz}$ when considering shear displacement.

As mentioned earlier, the shear displacement of the crush band is obstructed by the constraint wall soon after its initiation. The constraint exerts a force that is equal and opposite to F_{1-xz}, preventing further shearing (Fig. 2b). F_{2-z} (where $F_{2-z} = F_{1-xz} \cdot \sin\alpha$), one of the components of F_{1-xz}, will act in the direction opposite to F_z. In addition to F_{2-z}, the applied force also has to overcome the frictional resistance between foam surface and constraint wall in order to continue further deformation. The normal (to the constraint wall) component of

F_{1-xz} (i.e., $F_{3-x} = F_{1-xz} \cdot \cos\alpha$) contributes to the frictional force. If μ is the co-efficient of friction between foam surface and constraint wall, then the total upward force F_{up-z} (in the opposite direction of F_z) is given by

$$F_{up-z} = F_{2-z} + \mu \cdot F_{3-x}. \qquad (4)$$

This upward force gives rise to strain hardening. In the presence of F_{up-z}, the extra stress σ_z (in addition to σ_{p-z}) needed to continue deformation is F_{up-z}/a^2. The rate of change in σ_z with respect to the strain ε gives the strain hardening rate ($d\sigma_z/d\varepsilon$). This can be given by combining and simplifying Eq. 1 and Eq. 4 as follows

$$\frac{d\sigma_z}{d\varepsilon} = \frac{d}{d\varepsilon}\left[\left(\sigma_{p-z} \cdot \sin\alpha - \tau_{shear-xz} \cdot \frac{1}{\cos\alpha}\right) \cdot \sin\alpha + \mu \cdot \left(\sigma_{p-z} \cdot \sin\alpha - \tau_{shear-xz} \cdot \frac{1}{\cos\alpha}\right) \cdot \cos\alpha\right]. \qquad (5)$$

The reported value of μ is 0.3 for similar experimental conditions [3], in which the foam surface is considered flat during deformation. However, in the present case, μ is likely to differ as it is known that the dynamic friction coefficient decreases with increasing roughness [8]. It has already been shown that shear displacement results in uneven surfaces, Fig. 1b.

In case of constrained deformation, the range of α reported in the literature is 11−15° [1,4]. The following assumptions are made to estimate the strain hardening that results from the resistance against shear displacement. The value of $\tau_{shear-xz}$ is assumed to be $0.25 \cdot \sigma_{p-z}$ considering the maximum value (i.e., 15°) of α in Eq. 3 as explained earlier. It is also assumed that the contact between foam and constraint is flat-on-flat. Therefore, the value of $\mu = 0.3$ is used. The value of σ_{p-z} according to Ref. [1]

$$\sigma_{p-z} = 60.92 \cdot \left(\rho^*\right)^{1.5}, \qquad (6)$$

where ρ^* is the relative density of foam. It is assumed that for low values of strain (~0.05), when the first shear displacement occurs, there is only one crush band. Considering all these assumptions the value of $d\sigma_z/d\varepsilon$ was calculated from Eq. 5 as a function of ρ^* and was compared to the experimental data reported in Ref. [1]. According to Ref. [1], foams without constraint also exhibit strain hardening (see Fig. 3). This intrinsic strain hardening of unconstrained foams should be taken into account along with the calculated strain hardening to obtain the strain hardening in constrained foams. Accordingly, the strain hardening rates in constrained deformation were obtained by adding the calculated values of $d\sigma_z/d\varepsilon$ from Eq. 5 with the strain hardening rates of unconstrained deformation (see Fig. 3). Similarly, $d\sigma_z/d\varepsilon$ for $\alpha = 11°$ was calculated and is shown in Fig. 3. It is clear that $d\sigma_z/d\varepsilon$ changes significantly with α. For a low strain of 0.05 and $\alpha = 11°$ the amount of $d\sigma_z/d\varepsilon$ stemming from the resistance against shear displacement can give rise to sufficient strain hardening. The value is comparable to the reported strain hardening caused by a constraint in Ref. [1]. This is consistent with our previous findings: crush band's angles are more close to 11° than 15° [4]. The calculated strain hardening also shows, alike experimental data [1], that denser specimens exhibit greater hardening.

So far it has been assumed that the crush band is inclined only in the *x*-*z* plane. However, it is equally likely that the crush band can also be inclined in the *y*-*z* plane by any arbitrary angle, say β, as indicated by tomographic studies [4]. This is shown schematically in Fig. 4a where the crush band is indicated by a shaded region, it has a 3D geometry with flat surfaces. With increasing nominal load, shear displacement can take place in both planes. In constrained deformation both shear displacements will be prevented. In order to calculate the resultant upward force, one has to consider the prevention of shear displacement in both the planes separately. And then the resultant upward force (in the opposite direction of F_z) can be obtained by adding them up. In the presence of this kind of 3D crush band, strain hardening is likely to increase.

Some studies, and the present one, show that there can be multiple crush bands in monotonic loading [1,4,5], whereas only one crush band is reported in cyclic loading [4,9]. When multiple crush bands form, each band can have an arbitrary orientation. Some possible configurations of multiple bands are shown schematically in Fig. 4b. Crush bands can be inclined in the same direction as indicated by band A and B in Fig. 4b. Opposite orientations are also possible, see band B and C. Crush bands with opposite orientation can intersect each other inside the foam, see band D. Examples of multiple bands with arbitrary orientations have been reported in Ref. [4]. Shear displacement will take place in the same direction when the bands are inclined in the same direction. In contrast, shear displacements due to oppositely inclined crush bands will counteract each other. As multiple band formation is quite common in monotonic loading, one has to consider each band's orientation in order to calculate the effective load when lateral constraint is applied. Calculation of the force components is easier when the bands are considered flat. However, in practice, crush bands cannot be completely flat, it is rather corrugated. The direction of the forces on this kind of surfaces would vary from point to point. This would further make the calculations complex.

When the entire foam surface touches the constraint wall at the beginning of deformation, strain hardening can be calculated using the model suggested by Karthikeyan et al. [3]. In that case, a tri-axial stress state and friction between foam surface and constraint wall will be the main factors contributing to strain hardening. If the crush bands are perpendicular to the loading axis, the value of F_{up-z} will be zero, whereas when the crush bands are inclined to the horizontal axis and the force component along the inclination of the crush band is sufficient to cause shear displacement, F_{up-z} will have a finite positive value. If there is any gap between the foam surface and the constraint wall, the proposed mechanisms of strain hardening in Ref. [3] do not act until the foam surface comes into contact with the constraining wall. If deformation takes place as shear displacement, resistance against shear displacement and friction play a major role in strain hardening. Therefore, in order to estimate strain hardening, shear displacement should be taken into account in addition to the mechanisms proposed in Ref. [3].

In summary, it was shown that shear displacement in metal foams takes place, in both compression and fatigue loading, even in the presence of lateral constraint. The small gap between the foam surface and constraint wall provides the scope for shearing. Resistance against shear displacement was proposed as one of the mechanisms that cause strain hardening when the foam is subjected to deformation in the presence of a lateral constraint. Depending on the crush band's inclination angle, the amount of strain hardening rate can be significant and comparable to the experimental data reported in the literature.

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Figure captions

Fig. 1. X-ray radiographic images of two foams, (a) and (b) compression tested, (c) and (d) fatigue tested. (a) and (b) undeformed, (b) after 25.3%, and (d) after 13.7% plastic deformation in the presence of a lateral constraint. The face side of the foam aligned to the X-ray beam is marked by a broken box.

Fig. 2. (a) Orientation of a flat crush band inside foam before shear displacement. (b) Two-dimensional projected view of the same crush band after shear displacement, components of the nominal force and their direction along the crush band are shown. The arrow inside the crush band indicates the shear resistance of the foam. “d” indicates the gap between foam and constraint wall. (The arrows only indicate directions, not magnitudes)

Fig. 3. Experimental strain-hardening rate vs. relative density for foams tested with and without lateral constraint (data reproduced from Ref. [1]). The extra strain hardening stemming only from the resistance against shear displacement is added to the strain hardening of unconstrained specimens. Data shown for two different orientations of crush band. (con. = constrained, uncon. = unconstrained, exp. = experimental, calc. = calculated)

Fig. 4: Schematic of (a) 3-D shear displacement of a crush band which is inclined both in *x-z* and *y-z* plane, (b) multiple crush bands. Arrows indicate the possible direction of shear displacement of that particular crush band.

Figure 1a and 1b
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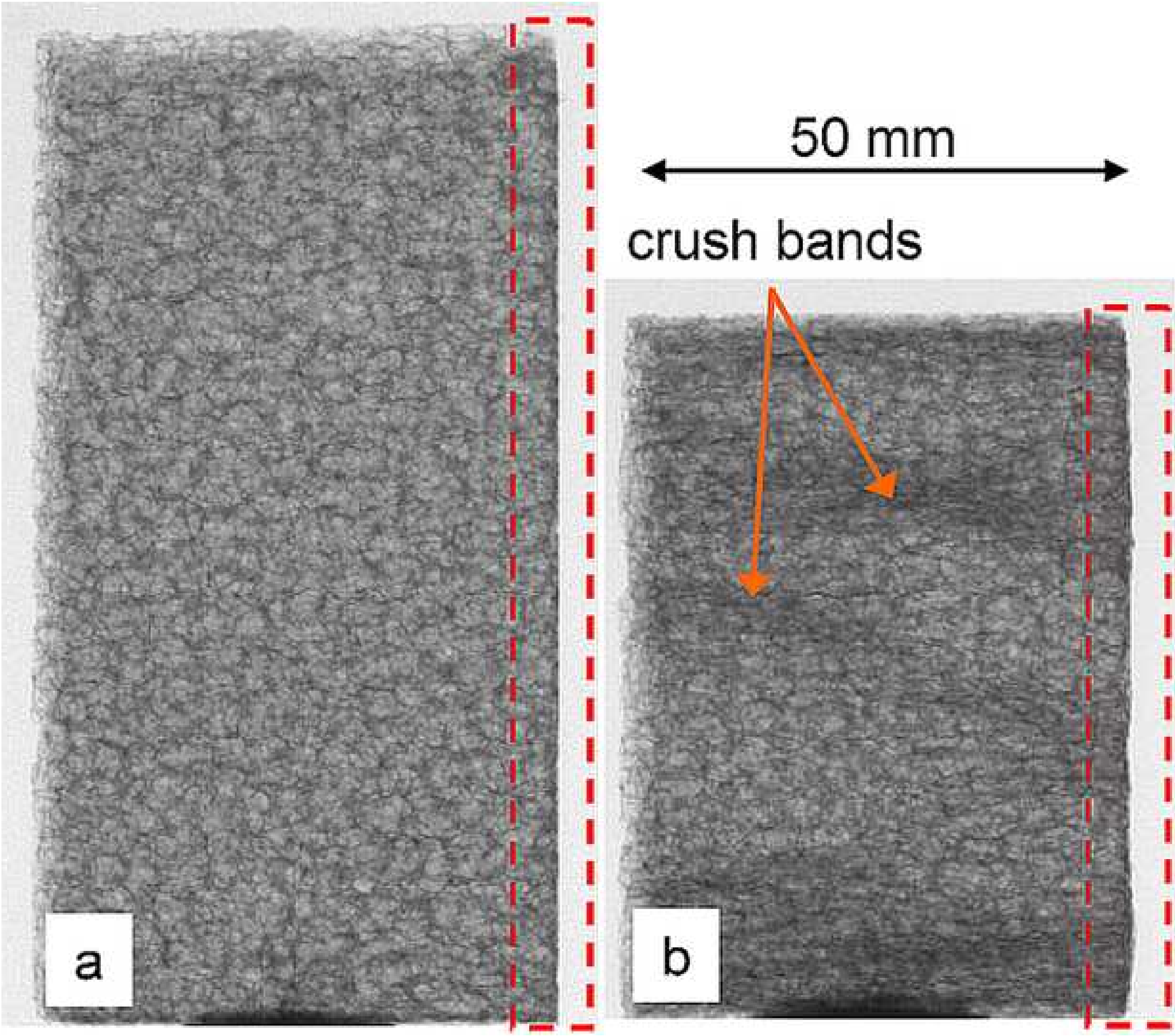

Figure 1c and 1d

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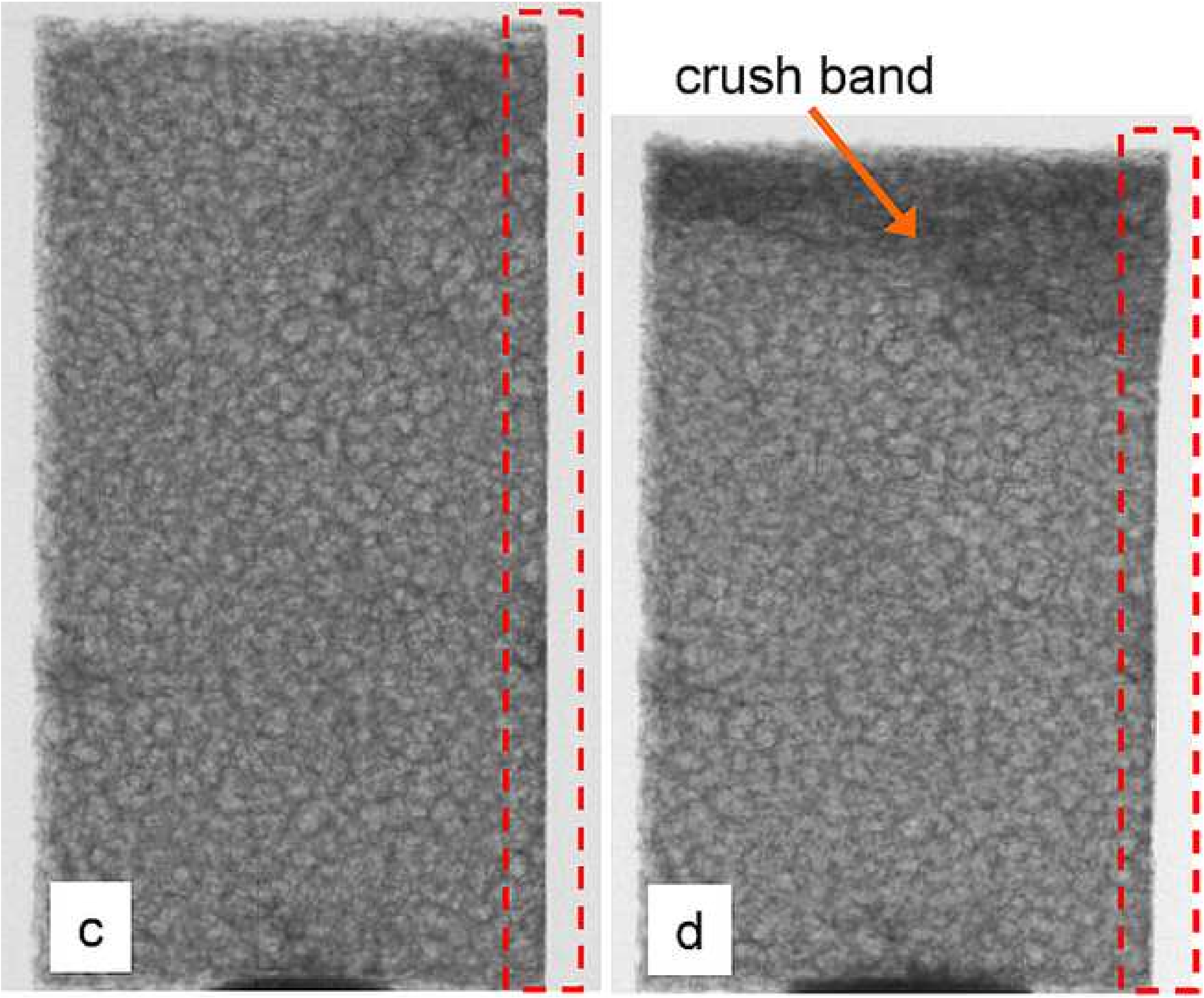

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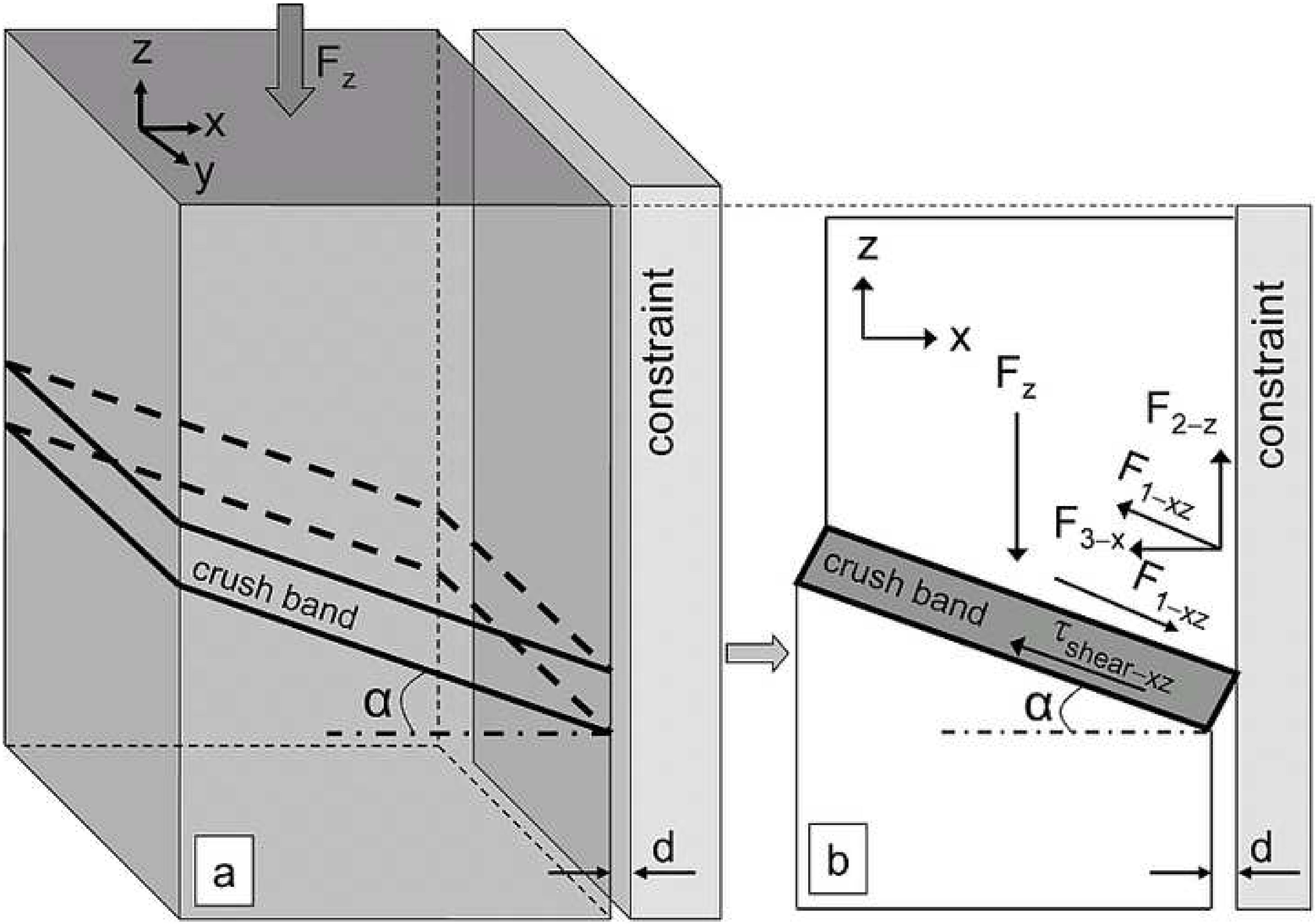

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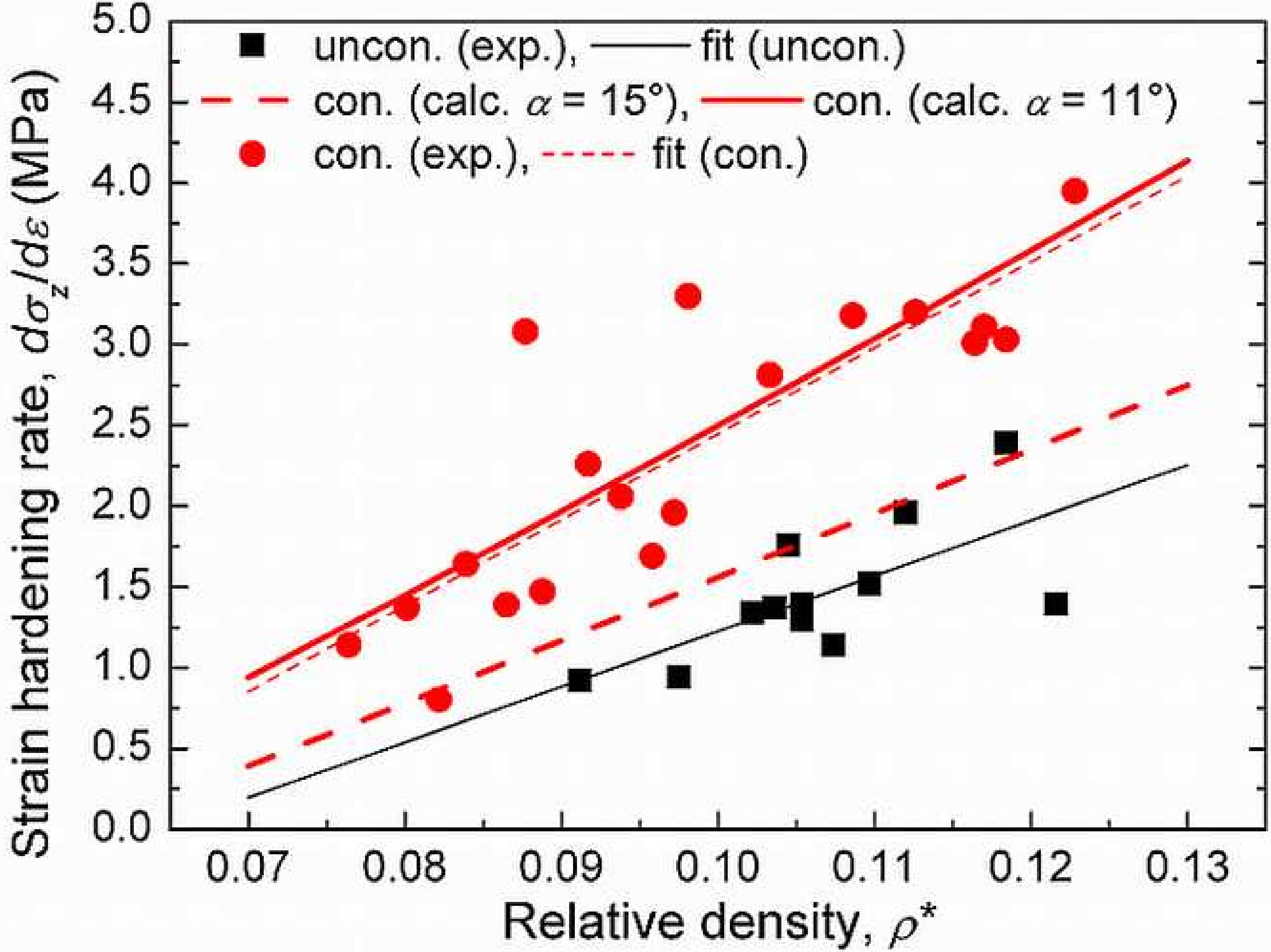

Figure 4
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