

ON THE OPTIMIZATION OF THE NON-LINEAR LATTICE OF BESSY III*

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Abstract

Helmholtz-Zentrum Berlin plans to construct a fourth-generation greenfield synchrotron light source in the early 2030s to replace BESSY II, a 1.7 GeV machine that has been running since 1998. The optimization of the linear lattice already considers non-linear aspects, such as minimizing the necessary sextupole strength and, for the minimal case of two families of sextupoles, phase cancellation to reduce the resonant driving terms. In preparation for the final optimization of 8 sextupole families and the single octupole, different approaches are compared: multi-objective genetic optimization, for a lattice with given error sets and orbit correction on the one hand, and the minimization of the resonant driving terms and the detuning terms on the other hand. Here, analytic formulas are used, so after a single evaluation of the Twiss parameters, the driving terms can be determined for different combinations of sextupole and octupole values. The results will determine the strategy for optimizing the lattice's non-linear behavior, i.e., dynamic aperture and momentum acceptance, taking the efficiency of the optimization into account.

INTRODUCTION

Using a novel deterministic approach to Multi-Bend-Achromat (MBA) lattice design [1], a baseline lattice has been defined for BESSY III [2]. As outlined in [3], the lattice is a 6-BA following the design criteria of higher-order achromats (HOAs), with distributed sextupoles. This approach controls the build-up of the resonant driving terms, RDTs, by local compensation of the sextupole contributions, for a scheme of two families of chromatic sextupoles (located at 8 symmetric positions). Some technically necessary adaptations were recently incorporated while still keeping the 16-fold symmetry. More space around the bending magnets for the beam absorbers was needed, and the distance between the defocusing quadrupole and the reverse bend in the unit cell was increased. To maintain the overall circumference, the space between the permanent magnet quadrupoles and the electromagnetic sextupoles was reduced from 100 to 60 mm, and the length of two quadrupoles was reduced. A single family of chromatic octupoles was introduced in the dispersion suppression cell at maximum dispersion and well-separated β -functions to handle momentum acceptance, see Fig. 1. These changes did not affect the non-linear performance of the lattice. In this paper, we compare different methods to optimize the non-linear elements in the lattice.

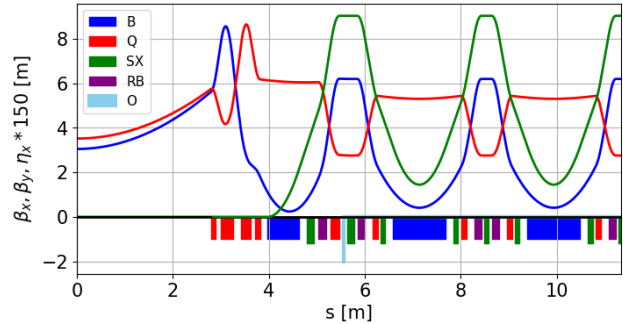


Figure 1: Half of the modified lattice of BESSY III. β_x : blue, β_y : red, η_x : green

MOGA OPTIMIZATION

Multi-objective genetic algorithms, MOGA, are a common choice to optimize many parameters for multiple goal functions. They have the advantage that complete physical models can be simulated, including errors, orbit correction, energy oscillation correlated with radiation, and more. Theoretically, they can represent the ultimate approximation to reality. Their drawback is the huge CPU and time consumption, even more so when not starting from a pre-optimized solution.

Table 1: Initial Standard Error Set

transverse misalignment (all magnets)	30 μm
roll (all magnets)	180 μrad
pitch and yaw - dipoles	60 μrad
pitch and yaw - other magnets	400 μrad
calibration	1e-4

The MOGA runs use the *NAGS-II* algorithm of the *Python pymoo* package [4]. The physics is covered by *pyAT* [5] and *pySC* [6]. The runs start from the linear lattice, with eight distinct, but symmetric, sextupole families, and a single octupole. Before optimization, the errors presented in Table 1 are applied to the lattice, and the orbit is corrected. For each evaluation, i.e., for each new set of multi-poles, if necessary, the orbit is re-corrected, and two predefined sextupole families correct for the chromaticity. Magnet values are confined to their technical limits. Full 6D tracking is performed for 1024 turns, more than one synchrotron period; insignificant further relative reductions of <0.1 of the MA were seen for 10.000 turns. Physical apertures of ± 9 mm are applied around the ring to avoid non-relevant results. The momentum acceptance (MA) and the area of the dynamic aperture (DA) in the center of the straight section are defined by particle loss. With a population of 300 points in the design space, each generation of the genetic algorithm produces 100 offsprings, and the optimization runs for 200 generations. For

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each generation, the population and the off-springs are evaluated according to their result in the objective space, and the 100 points with the worst results are discarded. A run takes around 16 hours on 100 CPUs of an HPC.

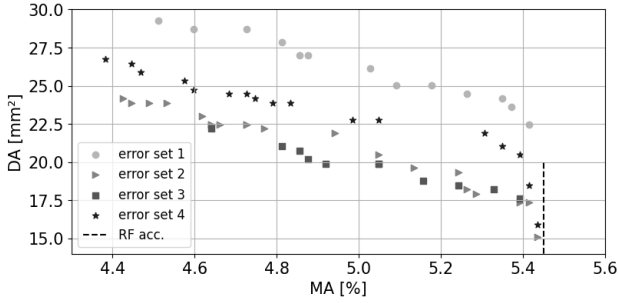


Figure 2: Pareto front for 3 different error sets.

Figure 2 shows the resulting Pareto fronts for four different error sets. The limit for the MA is $\approx 5.45\%$, which is the RF acceptance of the present design. All solutions show MAs $> 4.4\%$, and DAs up to 25 to 30 mm^2 , corresponding to a state of the art horizontal acceptance $A_x^2/\beta_x > 3.8 \text{ mm mrad}$.

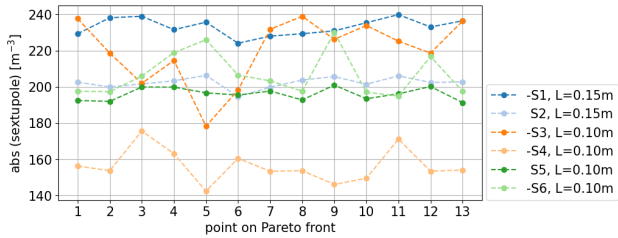


Figure 3: Absolute sextupole strength for error set 1.

Figure 3 shows the sextupole values related to the Pareto front of error set 1. The relative sigma for individual sextupole settings within a Pareto front is $< 8\%$ and their average is 4% . These are typical values, and the average sextupole values agree well between the different error sets. No correlation was found between the variation of individual sextupoles and DA, MA, or octupole value.

MINIMIZATION OF DRIVING TERMS

This work also used pyAT. All driving terms, DTs, can be expressed as analytic functions of the magnet data and the linear lattice functions, including $\delta\beta/\delta p$, $\delta\eta/\delta p$, and η' . Explicit formulas have been given, among others, in [7–9]. Linear lattice functions are independent of the values of the non-linear elements. Therefore, it is efficient to set up a data frame with all the necessary information once and work on this data frame during the optimization procedures.

The lower order DTs can be grouped into four sets: 1st order geometric and chromatic RDTs and 2nd order geometric (amplitude detuning) and 2nd order chromatic (momentum detuning) terms. The four sets of DTs have largely different magnitudes, but the size is comparable within each set. Therefore, the RMS value is used within each set to reduce the number of goal parameters. A weight function ([10],

Eq. 71) is introduced during the minimization process to level the different sizes between sets. The goal function is the sum of the RMS values of the four sets with additional weight factors to emphasize single groups.

In this paper, the "global" values of the DTs, i.e., their value at the end of the ring (or periodic structure), are considered, rather than the "DT fluctuations" proposed in [11]. Tests showed that global values converge much faster during minimization and that the results are comparable.

The selection of the minimization routine is critical. Minimization routines without constraints demand the nomination of two sextupole families for chromaticity compensation, and the result might depend on the choice of these families. Global minimization algorithms are too slow for repetitive tasks. The COBYLA algorithm [12] allows for inequality constraints suitable for adjusting the chromaticity $\xi_{x,y} < |1e^{-3}|$. COBYLA showed the fastest and most reliable performance of the different minimization routines tested.

Each minimization starts with the HOA baseline lattice, i.e., zero chromaticity, no errors, and the sextupoles are grouped into two types (SD, SF) of equal integrated strength. The variation limits cover the full technically possible range of each of the eight symmetric families. In each step, the DTs are derived, and the goal function is calculated. After convergence, the area of the DA and the MA are calculated for the optimal values. To mimic the limitations that errors would introduce, a reasonable limit of < 0.1 on the tune shift associated with amplitudes and momentum offsets is used.

Minimization: Tests

COBYLA is a local minimization algorithm, so several iterations are needed to avoid local dents. In the 8-dimensional sextupole space, comparable minima for different sextupole values could occur, but the iterations lead to convergence of the sextupole values; Two iterations proved sufficient when using global DTs terms.

The octupole plays an important role in the optimization of the non-linear behavior of the lattice. However, the RMS values of the 2nd order DTs are almost independent of the octupole value; the octupole contribution is linear in its strength, so that the x and y terms cancel each other to a good degree when building the RMS value. The octupole will not be "seen" by the minimization routine. Minimization has to be performed for distinct octupole values.

Small 1st and 2nd order DTs are a necessary condition for large DA and MA. But they are insensitive to the direction of the tune shift and the magnitude of 3rd or even higher-order terms. Therefore, a direct correlation between DA and MA is not granted.

Figure 4 shows the results when minimizing the goal function for increasing octupole strength and no weights. The geometric and chromatic parts of the goal function and the related DA and MA are shown. There is an encouraging correlation. While the MA varies between 3.6% and 4.3% , the DA loses 20% of its area within the technical limitations of the octupole strength.

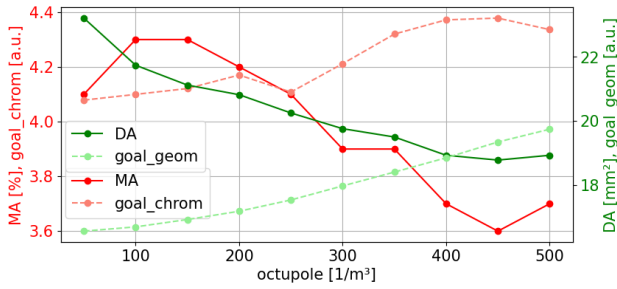


Figure 4: Opposite trend of the geometric DTs and the DA (green), and the chromatic DTs and MA (red).

Minimization: Results

For BESSY III, Touschek lifetime is of paramount importance to ensure operational flexibility across various user modes, such as TRIBs, VSR, and others.

Therefore, weights were applied to the 1st and 2nd order chromatic DTs. Figure 5 shows results. For each octupole value (colors), the weights range from 1, 5, 10 to 30 (growing dots). The results were clipped when the goal function exceeded that of the previous weight, indicating poorer convergence. For the resulting sextupole values, the DA and the MA were calculated using the tune shift limit of 0.1.

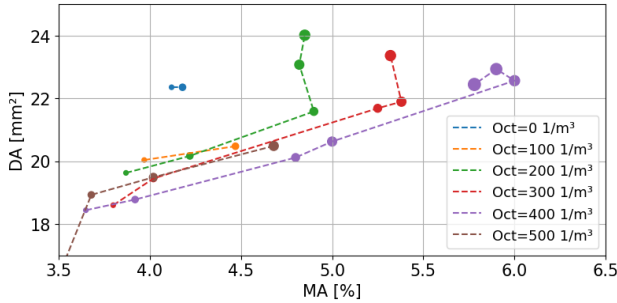


Figure 5: DA over the MA for different octupole strengths and increasing weights on the chromatic DTs (dot size).

For a constant octupole value, DA and MA increase with increasing weights. The best results, achieved between 200-400 1/m³, build a kind of 'Pareto front'. Although the octupole can increase the MA by almost 50 %, the DA exceeds that without the octupole at most by 7 %. The DA could not be improved either by different weights or by further optimization starting with the best results.

Figure 6 shows the related sextupole values (dashed lines). The relative sigma for individual sextupoles is <5.4 %. The average, 2.3 %, is almost half the MOGA value. For some sextupoles there is a slight correlation with the octupole strength that was not detected in the MOGA runs.

COMPARISON OF THE APPROACHES

The limit for the tune shifts used for calculating the DA and MA plays a critical role, since it mimics the distance to the nearest harmful resonance. Therefore, Fig. 7 displays the results for different tune shift limits of 0.1 (dots), 0.08

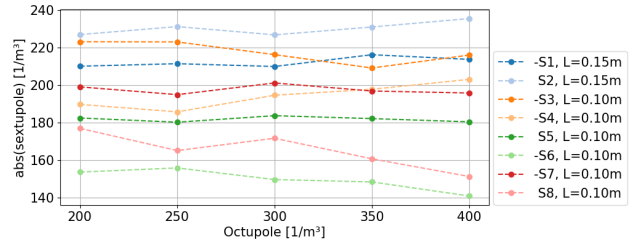


Figure 6: Absolute sextupole strength that achieve the lowest DTs for different octupole values .

(triangles), and 0.06 (squares). As expected, MA and DA diminish with tighter limits.

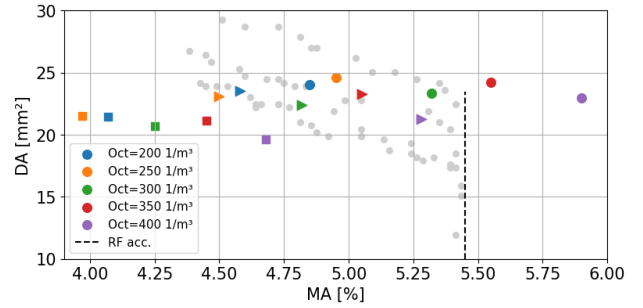


Figure 7: Results for tune shift limits of 0.1 (dots), 0.08 (triangles), and 0.06 (squares). Grey dots are MOGA results.

Figure 7 also displays the MOGA results (grey). The DT minimization results for tune shift limits of 0.08 and 0.1 agree well with the MOGA results, while a limit of 0.06 is too restrictive. MOGA calculations for certain error combinations might find DAs larger than predicted by DT minimization. However, there is no obvious benefit to MOGA versus DT minimization. Both methods result in a well-confined combination of sextupole strengths. Surprisingly, the combinations differ between the two methods, and both differ strongly from the initial HOA settings.

CONCLUSION

Minimizing DTs is an indirect approach that reduces the resonance width and the tune shifts of the particles. Minimization of the DTs depends on different minimization algorithm parameters and on weights. Although based on analytic formulas, a run for 7 weights (1-30) takes 70 min to 115 min on a laptop. DA and MA depend on the tune shift limit that is unknown a priori. This can be found by comparison with MOGA calculations. The Pareto front is subject to different error sets, resulting in ambiguities, on top of the amount of time and CPUs required.

The optimized DAs and MAs resulting from DT minimization with reasonable tune shift limits overlap with those calculated by MOGA. Despite that, the optimal sextupole settings of the two methods differ. The solution is not unique.

DT minimization can efficiently cover most of the workload of optimizing the non-linear lattice design, but few MOGA runs are needed as a reference.

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