Hamiltonian of the S=\frac{1}{2} dimerized antiferromagnetic-ferromagnetic quantum spin-chain BaCu$_2$V$_2$O$_8$

E. S. Klyushina,$^{1,2}$ A.T.M.N. Islam,$^1$ J. T. Park,$^3$ E. A. Goremychkin,$^4$ E. Wheeler,$^5$ B. Klemke,$^1$ and B. Lake$^{1,2}$

$^1$Helmholtz-Zentrum Berlin für Materialien und Energie, 14109 Berlin, Germany
$^2$Institut für Festkörperphysik, Technische Universität Berlin, 10623 Berlin, Germany
$^3$Heinz Maier-Leibnitz Zentrum, TUM München, 85747 Garching, Germany
$^4$Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX, United Kingdom
$^5$Institut Laue-Langevin, Boîte Postale 156X, F-38042 Grenoble Cedex 9, France

The novel quantum magnet BaCu$_2$V$_2$O$_8$ was recently discovered to be a rare physical realization of a one-dimensional antiferromagnetic - ferromagnetic dimerized chain which displays strongly correlated phenomena at elevated temperatures [Phys. Rev. B 93 241109(R) (2016)]. This current paper presents an extended study of the Hamiltonian of BaCu$_2$V$_2$O$_8$ at base temperature. Static susceptibility and inelastic neutron scattering data are compared to several theoretical models. An analytical relation for the dynamic structure factor of the complex unit cell of BaCu$_2$V$_2$O$_8$ is derived and used to identify the intrachain exchange paths. Further analysis using the first moment of the dynamic structure factor was employed to determine the exchange path responsible for the intradimer interaction. This analysis reveals that the dimer chain is formed by a dominant antiferromagnetic exchange interaction $J_{\text{intra}}$=40.92 meV which is realized via the Cu-O-V(II)-O-Cu super-exchange path and a weak ferromagnetic coupling $J_{\text{inter}}$=-11.97 meV which arises within the copper-oxygen double-plaquettes.

I. INTRODUCTION

Dimerized quantum magnets have attracted considerable attention in the last decades since extraordinary quantum phenomena such as bound- and multi-magnon states, Bose-Einstein Condensation, and strongly correlated behavior at finite temperatures were discovered in these systems. In the dimerized magnetic system, spin-half magnetic ions are strongly coupled by the antiferromagnetic (AFM) exchange interaction into pairs known as dimers. These dimers are characterized by a spin-singlet non-magnetic ground state and degenerate S=1 triplet excitations which are gapped. The excitations are called magnons or triplons and are restricted by the hard-core constraint which allows only one excitation per dimer site. Dimers interact with each other via this hard-core repulsion and via weak interdimer exchange interactions which couple them in one- two- or three-dimensional structures. In one-dimensional (1D) dimerized quantum magnets the strong intradimer magnetic exchange interaction $J_{\text{intra}}$ alternates with a weaker interdimer magnetic exchange coupling $J_{\text{inter}}$ along one particular direction within the crystal lattice so that the dimers form dimerized chains. The Hamiltonian of the dimerized chain is

$$H = J_{\text{intra}} \sum_{i=1} \vec{S}_{2i-1} \cdot \vec{S}_{2i} + \alpha \vec{S}_{2i} \cdot \vec{S}_{2i+1} \quad (1)$$

Here, $\vec{S}_{2i}$ is the spin of the magnetic ion at the 2i crystal site and $\alpha = J_{\text{inter}} / J_{\text{intra}}$ is the dimerization ratio where the positive sign of $J_{\text{intra}}$ and $J_{\text{inter}}$ defines an AFM interaction and a negative sign a ferromagnetic (FM) interaction. Dimerized chains are particularly interesting because they can develop strongly correlated behaviour at finite temperatures which is caused by strong interactions between the excitations via the interdimer exchange coupling and the hard-core repulsion. The strongly correlated phenomena manifest themselves as an asymmetric Lorentzian thermal line shape broadening of the magnetic excitations and was first observed in the AFM- AFM dimerized chain compound copper nitrate. This phenomena is not limited to 1D systems and was detected more recently in the three-dimensional (3D) dimerized antiferromagnet Sr$_3$Cu$_2$O$_8$. Although dimerized quantum antiferromagnets have been intensively studied, dimerized magnetic systems which have ferromagnetic interdimer exchange coupling have not been well explored. This is because physical realization of AFM-FM dimerized quantum magnets are very rare, and in most cases their Hamiltonians have not been accurately determined via inelastic neutron scattering. Indeed, only a few inorganic compounds have been suggested as physical realizations of the S=1/2 dimerized chain with a high magnetic energy scale and which displays strongly correlated behavior at elevated temperatures similar to that observed in several other dimerized compounds. BaCu$_2$V$_2$O$_8$ is particular interesting...
FIG. 1. (a) The crystal structure of BaCu$_2$V$_2$O$_8$. For clarity only two of the four chains are shown and the Ba$^{2+}$ ions are omitted. The blue and gray filled circles represent the magnetic Cu$^{2+}$ ions and oxygen O$^{2−}$ anions, respectively. The dark and light green filled circles are the non-magnetic V$^{5+}$ (I) and V$^{5+}$ (II) vanadium ions which have different crystallographic sites, respectively. The dashed gray lines give the borders of the unit cell. (b) Projection of the crystal structure of BaCu$_2$V$_2$O$_8$ on the ab-plane where the blue circles indicate the direction of rotation of the screw-chains along the c-axis. (c-d) Model 1 and Model 2 for the magnetic exchange paths in BaCu$_2$V$_2$O$_8$ discussed in the text and suggested (c) by Z. He et al. and (d) by H.-J. Koo et al. respectively. The dashed red, solid yellow and double red lines represent the Cu-O-V(I)-O-Cu (J$_1$), Cu-O-Cu (J$_2$) and Cu-O-V(II)-O-Cu (J$_3$) magnetic exchange paths respectively. $d_1$, $d_2$ and $d_3$ give the direct distances between the magnetic Cu$^{2+}$ ions in the exchange paths J$_1$, J$_2$ and J$_3$. The $D_{model1}$ and $D_{model2}$ give the dimer periodicity for Model 1 and Model 2.

because its magnetic excitations retain their coherence up to T=250K which is much higher than the temperatures reported for other dimerized compounds where strongly correlated behavior was also observed. This makes BaCu$_2$V$_2$O$_8$ an model compound and detailed knowledge of its Hamiltonian is important for a better understanding of its magnetic properties.

BaCu$_2$V$_2$O$_8$ has a tetragonal crystal structure with the space group I42d (lattice parameters a=b=12.744 Å, c=8.148 Å) where the S=1/2 magnetic Cu$^{2+}$ ions are coordinated by O$^{2−}$ and form square-planar CuO$_4$ plaquettes (Fig. 1). These CuO$_4$ plaquettes are paired into edge-sharing Cu$_2$O$_6$ double-plaquettes. These double plaquettes are connected to each other via VO$_4$ tetrahedra with non-magnetic V$^{5+}$ ions, and form two-fold screw chains along the c-axis (Fig. 1(a)). The c-axis always lies within the Cu$_2$O$_6$ double plaquettes which rotate about it by 180°. There are 4 screw chains within each unit cell, two of which are left-handed and two are right-handed where the left-handed screw chains are rotated by 90° with respect to the right-handed screw chains (Fig. 1(b)).

The magnetic models of BaCu$_2$V$_2$O$_8$ proposed in the literature vary from the dimerized AFM chains to isolated AFM dimers. First, Z. He et al. suggested that BaCu$_2$V$_2$O$_8$ consists of non-interacting pairs of linear dimerized chains which arise along the c-axis via the dominant AFM superexchange path Cu-O-V(I)-O-Cu (path J$_1$) and a weaker AFM superexchange path Cu-O-Cu (path J$_2$) within the edge-sharing double plaquettes (Fig. 1(c)). In this model each crystallographic screw chain gives rise to one pair of linear magnetic chains, and each magnetic chain has one dimer per c lattice parameter. We call this Model 1. The values of the energy gap ($\Delta$), dominant intradimer exchange coupling ($J_{\text{intra}}$) and dimerization ratio (\(\alpha\)) were estimated to be $\Delta=230$ K, $J_{\text{intra}}=260$K and $\alpha=0.2$ from the analysis of powder DC magnetic susceptibility data. Later, the results of $^{51}$V nuclear magnetic resonance measurements suggested that $\Delta=450$K-470K within the same model which is consistent with the value obtained by Z. He et al. The factor of 2 between these results is due to the fact that Z. He et al. used a Hamiltonian whose parameters are rescaled by 2 compared to Eq (1).

Another model was developed by H.-J. Koo et al. and based on the results of extended Hückel tight-binding calculations. They suggested that BaCu$_2$V$_2$O$_8$ consists of dimerized AFM chains which run along the c-axis, but in contrast to the first model, these chains are not straight but are in fact the screws chains evident in the crystal structure with two dimers per c lattice parameter (Fig. 1(d)). We call this Model 2. The calculations predicted a dominant AFM super exchange path Cu-O-V(II)-O-Cu (path J$_3$) and a weaker AFM path J$_2$. The dimerization ratio is estimated to be $\alpha=0.164$ assuming a priori that the magnetic exchange constants are AFM.

First principles electronic structure (FPES)
exchange path to form screw-chains along the c-axis. Bounded ferromagnetically via the Cu-O-Cu magnetic 1D AFM-FM system where the AFM dimers are weakly calculated by H.-J. Koo et al. However, the FPES-calculations estimated the dimerization ratio to be α=0.05 which is much smaller than α=0.164 calculated using the Hückel tight-binding approach. Thus, according to the FPES-calculations BaCu$_2$V$_2$O$_8$ can be viewed as an isolated dimer system rather than a dimerized chain.

We recently explored the magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ at base temperature and compared it to the results of state of the art numerical computations performed using the DMRG technique. This comparison revealed that BaCu$_2$V$_2$O$_8$ is an AFM-FM screw-chain where the AFM dimers are coupled by weak ferromagnetic interdimer interactions along the c-axis. This is in contrast to all previous results which suggest that all interactions in BaCu$_2$V$_2$O$_8$ are AFM. Although the Hamiltonian of BaCu$_2$V$_2$O$_8$ was solved, the magnetic exchange path responsible for the dominant AFM coupling was not identified nor was the relation for the complex dynamic structure factor observed experimentally at base temperature derived analytically.

This paper presents a detailed investigation of the Hamiltonian of BaCu$_2$V$_2$O$_8$ using static magnetic susceptibility and inelastic neutron scattering (INS) measurements whose results were analyzed (i) to identify the dominant magnetic exchange path and (ii) to introduce the analytical relation for the complex dynamic structure factor of BaCu$_2$V$_2$O$_8$ observed experimentally at base temperature.

The paper is organized as follows. First, we carefully explore the static susceptibility and magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ to solve the Hamiltonian of this compound. After that, we derived the analytical relation for the dynamic structure factor of the dimerized screw chain which allows the complex magnetic excitation spectrum of BaCu$_2$V$_2$O$_8$ to be computed using the extracted Hamiltonian. Finally, we analyze the intensity distribution of the magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ using the relation for the first moment of the dynamic structure factor to identify the dominant AFM exchange path.

In this paper, we prove that BaCu$_2$V$_2$O$_8$ is a dimerized 1D AFM-FM system where the AFM dimers are weakly ferromagnetically bound via the Cu-O-Cu magnetic exchange path to form screw-chains along the c-axis.

II. EXPERIMENTAL DETAILS

Powder and single crystalline samples of BaCu$_2$V$_2$O$_8$ were grown in the Core Lab for Quantum Materials at the Helmholtz Zentrum Berlin fur Materialien und Energie (HZB) in Germany. The polycrystalline powder sample and feed rod for single crystal growth of BaCu$_2$V$_2$O$_8$ were both prepared by standard solid state reactions using high purity powders of BaCO$_3$ (99.997%, Alfa Aesar), CuO (99.995%, Alfa Aesar) and V$_2$O$_5$ (99.99%, Alfa Aesar). After mixing thoroughly in ethanol, in the 1:2:1 molar ratio, powder of stoichiometric composition was sintered in air at 650°C twice for 12 hours with intermediate grinding. The powder was then packed in a rubber balloon and pressed hydrostatically up to 3000 bars in a cold-isostatic-pressure (CIP) machine and sintered in air at 750°C for 12 hours to form a dense cylindrical rod of about 6 mm in diameter and about 7 cm in length.

The crystal growth of the single crystalline samples of BaCu$_2$V$_2$O$_8$ were performed in a four mirror type floating-zone machine (Crystal Systems Corp., Japan) equipped with Tungsten halide lamps (Crystal Systems Corp., Japan). Since BaCu$_2$V$_2$O$_8$ is known to decompose around 770°C close to the melting temperature, crystal growth was done by the traveling-solvent-floating-zone (TSFZ) technique using a small amount (0.5~g/m) of off-stoichiometric solvent having 5mol% excess V$_2$O$_5$, prepared by solid state reactions as for feed rod. Growth was performed in ambient air atmosphere at a speed of 0.2 mm/hour. The as-grown single crystal was about 4.5 cm in length and about 5.5 mm in diameter. After growth, a small piece of the single crystal was ground and checked with X-ray powder diffraction (Bruker D8) for phase purity. The single crystallinity of the sample was confirmed by systematically taking several X-ray Laue patterns from different parts of the crystal.

To explore the bulk properties of BaCu$_2$V$_2$O$_8$, static magnetic susceptibility measurements were performed on two single crystalline samples of BaCu$_2$V$_2$O$_8$ with masses of 19.9 mg and 19.2 mg which were cut from different single crystal growths. The measurements were carried out at the Core Lab for Quantum Materials, HZB, using a Physical Property Measurement System (PPMS) and were performed under a magnetic field of 1T applied both parallel and perpendicular to the c-axis. The data were collected at both low (2 K-400 K) and high (300 K-900 K using a heater stick) temperature regimes. Another measurement without the sample but with the same quantity of glue was also performed for the two temperature ranges to give an estimate of the background, this was fitted and subtracted from the data. For further analysis the data collected in the low- and high-temperature regimes were normalized and combined by averaging over the overlapping temperature range (300 K-400 K).

The magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ were explored using both powder and single crystal inelastic neutron scattering techniques. Powder INS data were collected for a polycrystalline sample of BaCu$_2$V$_2$O$_8$ with a total mass of 21~g using the high count rate thermal time-of-flight spectrometer MERLIN at ISIS, Rutherford Appleton Laboratory, U.K. The sample was cooled to a base temperature of 5 K using a closed cycle cryostat and the data were collected at both base and room temperatures (T=300K) to distinguish the magnetic contri-
bution from the phonons. The Fermi chopper was operated in two different modes characterized by frequencies of 250 Hz and 300 Hz and was phased to select neutrons with incident energies of $E_i=36 \text{meV}$ and $E_i=50 \text{meV}$, respectively. The corresponding resolution widths at zero energy transfer were found to be 2.2 meV and 3.8 meV, respectively.

The single crystal magnetic excitation spectra of BaCu$_2$V$_2$Os were explored by INS measurements performed on both the thermal triple-axis spectrometer PUMA operated by the Heinz Maier-Leibnitz Zentrum, Garching, Germany and on the high-flux thermal neutron triple-axis spectrometer IN8 at the Institut Laue-Langevin (ILL) in Grenoble, France.

For the measurements on PUMA, a single crystal sample of BaCu$_2$V$_2$Os with a mass of 2.16 g was preoriented with the $(h,0,l)$ scattering plane and mounted in a closed-cycle cryostat. The data were collected at a base temperature of 3.5 K using two different experimental settings. First with a double-focussed pyrolytic graphite monochromator (PG (002)) together with a doubly-focussed PG (002) analyzer which provided a calculated resolution of 2 meV at a fixed final wave-vector of $k_f=2.662 \text{Å}^{-1}$. Then, the PG (002) monochromator was changed to the double-focused single crystal Cu monochromator (Cu(220)) which is characterized by a calculated energy resolution value of 0.74 meV at fixed final wave vector of $k_f=1.97 \text{Å}^{-1}$ and 0.992 meV at fixed final wave vector of $k_f=2.662 \text{Å}^{-1}$.

A smaller crystal of BaCu$_2$V$_2$Os (from the same growth) with a mass of 1.38 g was used for the INS measurements at the IN8 instrument. The sample was preoriented with the $(h,h,l)$ plane in the instrumental scattering plane and mounted into the small orange cryostat which allows it to be cooled down to a base temperature of $T=1.5 \text{K}$. A bent perfect crystal silicon monochromator (bent Si (111)) was used together with a bent Si (111) analyzer operated at $k_f=2.662 \text{Å}^{-1}$ and giving an energy resolution of 1.07 meV. The main advantage of a silicon analyzer is that it has no second order Bragg peaks which reduces the background contribution.

**III. RESULTS AND ANALYSIS**

**A. Static Susceptibility**

Figure 2 shows the background subtracted and normalized DC susceptibility data collected on a single crystalline sample of BaCu$_2$V$_2$Os over the temperature range from 2 K to 900 K. The constant magnetic field of 1 T was applied both parallel ($\chi_{H//c}$) and perpendicular ($\chi_{H//c}$) to the c-axis. The data reveal isotropic behavior within the whole temperature range except for the difference of the amplitudes of $\chi_{H//c}$ and $\chi_{H//c}$. This difference is attributed to the anisotropy of the $g$-factor which will be discussed below.

Both $\chi_{H//c}$ and $\chi_{H//c}$ show a broad maximum in the vicinity of $T=280 \text{K}$ which is commonly observed in low-dimensional quantum magnets. Below this maximum, $\chi_{H//c}$ and $\chi_{H//c}$ smoothly decrease as temperature is lowered and do not display any signatures of a phase transition. At very low temperatures ($T<15 \text{K}$) both data sets start to increase slightly as temperature tends to zero. This behavior is associated with the paramagnetic contribution from the impurities which starts to play a role only at the lowest temperatures due to a small quantity of these impurities. Apart from this low temperature impurity tail, $\chi_{H//c}$ and $\chi_{H//c}$ tend towards to zero at zero temperature suggesting that this system has a non-magnetic ground state and gapped magnetic excitation spectra. Both $\chi_{H//c}$ and $\chi_{H//c}$ could be fitted over the whole temperature range using the weakly-coupled dimer model and taking into account the impurity contribution as well as the diamagnetic susceptibility and van Vleck paramagnetism:

\[
\chi_{\text{fit}} = \chi_0 + \chi_{VV} + \chi_{\text{imp}} + \chi_{\text{wed}}
\]

Here, $\chi_0$ and $\chi_{VV}$ describe the temperature independent diamagnetic core susceptibility and van Vleck paramagnetism, respectively.

The $\chi_{\text{imp}}$ describes the paramagnetic Curie tail observed in the data at temperatures below 15 K which can be fitted using the Curie-Weiss law:

\[
\chi_{\text{imp}} = \frac{C_{\text{imp}}}{T - \Theta_{\text{imp}}}
\]
where $C_{\text{imp}}$ and $\Theta_{\text{imp}}$ are the Curie constant and Curie temperature of the impurity, respectively. The last term $\chi_{\text{wcd}}$ represents the weakly-coupled-dimer mode which describes the magnetic susceptibility of weakly coupled dimers within the mean field theory:

$$\chi_{\text{wcd}} = \frac{N_A g^2 \mu_B^2 S(S + 1)/k_B T}{3 + \exp(J_{\text{intra}}/k_B T) + J_{\text{inter}}/k_B T}$$

Here, $N_A$ is Avogadro number, $g$ is the Landé-factor, $\mu_B$ is the Bohr magneton, $S=1/2$ is the spin value. The weakly coupled-dimer model does not assume any particular dimensionality of the magnetic system but allows $J_{\text{intra}}$ and $J_{\text{inter}}$ to be estimated. This model is found to be appropriate to describe the magnetic properties of BaCu$_2$V$_2$O$_8$ because the susceptibility data suggest that BaCu$_2$V$_2$O$_8$ has a gapped non-magnetic ground state without long-range magnetic order.

The solid red lines through $\chi_{H||c}$ and $\chi_{H\perp c}$ in Fig. 2 present the best fits which were achieved using relation (4) with the values of $J_{\text{intra}} = 39.67(7) \pm 0.003$ meV, $J_{\text{inter}} = -12.52 \pm 0.07$ meV, $g_{H||c} = 2.09 \pm 0.03$ for $\chi_{H||c}$ and $J_{\text{intra}} = 39.939 \pm 0.003$ meV, $J_{\text{inter}} = -7.23 \pm 0.07$ meV, $g_{H\perp c} = 2.27 \pm 0.03$ for $\chi_{H\perp c}$. The values of exchangecouplings extracted from each other and can be averaged to $J_{\text{intra}} = 39.80 \pm 0.13$ meV and $J_{\text{inter}} = -9.87 \pm 0.64$ meV. These results imply that BaCu$_2$V$_2$O$_8$ contains of antiferromagnetic (AFM) dimers which are weakly coupled ferromagnetically (FM) with the dimerization ratio of $\alpha = 0.248$.

### B. Magnetic excitation spectrum

#### 1. Powder inelastic neutron scattering

To get an overview of the magnetic excitation spectrum of BaCu$_2$V$_2$O$_8$, powder inelastic neutron scattering (INS) measurements were performed using the MERLIN time-of-flight spectrometer. Figure 3(a) shows the powder INS data of BaCu$_2$V$_2$O$_8$ collected at the base temperature of $T=5$ K and plotted as a function of wavevector ($|\vec{Q}|$) and energy (E) transfers. These data reveal that BaCu$_2$V$_2$O$_8$ has a gapped magnetic excitation band which extends over the energy range of $\approx$35-47 meV. The intensity of these excitations is strong at low wavevectors and decreases with increasing wavevector confirming their magnetic nature which was also verified by the comparison with the high temperature data. Indeed, Figure 3(b) shows the INS data on the same sample at $T=300$ K which reveals that the intensity of the excitations above 30 meV decreases with increasing temperature, proving their magnetic origin. The signal below 30 meV, in contrast, becomes more intense with temperature confirming that these are phonons. The strong temperature independent signal at low energies of 0-5 meV on both plots is incoherent elastic scattering.

Figure 3(c) shows the cuts through the INS data collected at $T=5$K (green squares) and $T=300$K (blue diamonds) integrated over the range of $3.8 \leq |\vec{Q}| \leq 4$ $\AA^{-1}$ and plotted as a function of the energy transfer. The low temperature data clarify that the magnetic excitation band extends over the range of 36.1-45.1 meV and is centered at $E_0 \approx 40.5$ meV. The band width is extracted to be $\approx$9 meV and the ratio of the energy gap to the band width is $\approx 4$ revealing that this is a strongly dimerized magnetic system. The high temperature data reveal that the signal becomes weaker with increasing temperature proving its magnetic origin.

#### 2. Single crystal inelastic neutron scattering

Single crystal INS measurements of BaCu$_2$V$_2$O$_8$ were performed using the thermal neutron triple-axis spectrometer PUMA at the base temperature of $T=3.5$ K to further explore the magnetic excitations spectrum of this compound. The measurements were performed within the $(h,0,l)$ scattering plane either along the $l$ or $h$ directions to investigate the dispersion of the magnetic exi-
tations along the main crystal axes.

Figure 3(a)-(d) shows the magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ measured along the (6,0,l), (5,0,l), (3,0,l) and (2,0,l) directions, respectively. The results reveal that there are two gapped excitations branches dispersing over the energy range of $\approx 36$ meV-46 meV. Both modes have the same periodicity and bandwidth but are shifted with respect to each other by a half period and alternate in intensity. Indeed, the mode, which has an intense minimum at even-integer of the expansion and the coefficients $a_n$ proposed in the literature for the dimerized chain, implies that there are two flat modes at $E \approx 36$ meV and $E \approx 46$ meV whose intensities alternate along $l$ direction.

In contrast to (h,0,4), only one mode has an intensity along the (h,0,5) direction where $h$ varies from $h=0$ to $h=6$. This is in agreement with the intensity distribution observed along (3,0,l) direction. The absence of any dispersion of the modes along the $h$-direction implies that the coupling between the chains within the ab-plane is negligible.

The observed magnetic excitation spectra suggest that BaCu$_2$V$_2$O$_8$ is a dimerised chain where dimers are coupled one-dimensionally (1D) along the c-axis ($l$ direction in the reciprocal space). The fact that the two modes are identical but shifted with respect to each other by a half of period along the $l$-direction, implies that this chain is not straight and contains two dimers per magnetic unit cell giving rise to zone folding.

C. Hamiltonian of BaCu$_2$V$_2$O$_8$

To learn more about the dimer exchange path and the magnetic exchange constants, the magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ was analyzed using the 5th order expansion of the one-magnon dispersion relation $\omega(q,h,l)$ proposed in the literature for the dimerized chain,

$$\omega(q,h,l) = J_{\text{intra}} \cdot \sum_{n=0}^{5} a_n(\alpha) \cdot \cos(n \cdot q_l \cdot D) \quad (5)$$

Here, $q_l = \frac{2\pi l}{c}$ is the wave vector transfer along the chain direction which is the $l$ direction for BaCu$_2$V$_2$O$_8$; $D$ is the separation between dimers along the c-axis; $n$ is the integer of the expansion and the coefficients $a_n$ are,

$$a_0 = 1 - \frac{1}{16} \cdot \alpha^2 + \frac{3}{64} \cdot \alpha^3 + \frac{23}{1024} \cdot \alpha^4 + \frac{3}{256} \cdot \alpha^5$$
$$a_1 = \frac{1}{2} \cdot \alpha - \frac{1}{4} \cdot \alpha^2 + \frac{1}{32} \cdot \alpha^3 - \frac{5}{256} \cdot \alpha^4 - \frac{35}{2048} \cdot \alpha^5$$
$$a_2 = \frac{1}{16} \cdot \alpha^2 - \frac{1}{32} \cdot \alpha^3 - \frac{15}{512} \cdot \alpha^4 - \frac{283}{18432} \cdot \alpha^5$$
$$a_3 = -\frac{1}{64} \cdot \alpha^3 - \frac{1}{48} \cdot \alpha^4 - \frac{9}{1024} \cdot \alpha^5$$
$$a_4 = -\frac{5}{1024} \cdot \alpha^4 - \frac{7}{9216} \cdot \alpha^5$$
$$a_5 = -\frac{1}{4096} \cdot \alpha^5 \quad (6)$$

The comparison of the energy dispersion observed in the magnetic excitation spectra of BaCu$_2$V$_2$O$_8$ along the $l$ direction with $\omega(q)$ can be used to obtain $D$. Indeed, the INS data collected along the (6,0,l), (5,0,l), (3,0,l), (2,0,l) directions show that each mode disperses along $l$ with a periodicity of $P = 2$ r.l.u. At the same time, the first
FIG. 4. Background subtracted single-crystal INS data measured along the (a) (6,0,1), (b) (5,0,1), (c) (3,0,1), (d) (2,0,1) directions using the PUMA spectrometer at base temperature. The black dots indicate the points of measuring. The white arrows show the directions of the constant wavevector scans measured at (6,0,1) and (6,0,2) as discussed in the text, the colors give the relative intensity. The DSF computed along the (e,i) (6,0,1), (f,j) (5,0,1), (g,k) (3,0,1), (h,l) (2,0,1) directions for AFM-FM and AFM-AFM Hamiltonian using \( S_{sd}(q, \omega)(eq. (7)) \), respectively.

non-constant term in the expansion suggests that the period of the dispersion in the dimerized chain is given by \( P = \frac{2 \pi}{c^2} \). Thus, the dimer repeat distance is immediately found to be half of the \( c \)-lattice parameter.

The extracted value of \( D = \frac{c}{2} \) implies that there are two dimers per \( c \) lattice parameter for every chain. This is only possible for Model 2 (Fig1 (d)) where the magnetic chains are in fact the screw chains. Model 1 in contrast gives rise to linear chains where there is only one dimer per \( c \) lattice parameter (Fig1 (c)).

Model 2 suggests that the magnetic exchange couplings are realized via the \( J_2 \) path within the double-plaquettes and the \( J_3 \) path between the plaquettes. These two exchange paths form screw chain by alternating twice along the \( c \)-axis within one crystallographic unit cell leading to a reduction of the Brillouin zone by a factor of 2 along the \( l \) direction and the appearance of the second mode in the excitation spectrum.

The presence of two modes can be demonstrated by calculating the dynamical structure factor (DSF) for the magnetic screw-chains using the method outlined in Ref. and taking into account that the chains are dimer-
FIG. 5. Background subtracted single-crystal INS data measured along the (a) \((h,0,5)\), (b) \((h,0,4)\) directions using PUMA spectrometer at base temperature. The black dots indicate the points that were measured and the colors give the relative intensity. The DSF computed along the (c,e) \((h,0,5)\), (d,f) \((h,0,4)\) directions for AFM-FM and AFM-AFM Hamiltonian using \(S_{sd}(q,\omega)\)(eq. (7)), respectively.

ized. Using the mathematical transformations described in Appendix A the DSF \(S_{sd}(q,\omega)\) for the unit cell of \(\text{BaCu}_2\text{V}_2\text{O}_8\) is given by:

\[
S_{sd}(q_{h,k,l},\omega) = [\cos^2(\psi_1) + \cos^2(\psi_2)] \cdot S_d(q_{h,k,l},\omega) + (\sin^2(\psi_1)+\sin^2(\psi_2)) \cdot S_d(q_{h,k,l+1},\omega) \cdot |F_{anis}(q_{h,k,l})|^2
\]  

\tag{7}

with

\[
\psi_1 = q_h \cdot \Delta_{2(3)}; \quad \psi_2 = q_k \cdot \Delta_{2(3)}
\]  

\tag{8}

where \(\omega\) is the frequency and \(q_{h,k,l}\) is the wavevector transfer, \(\Delta_{2(3)} \approx 0.09\cdot a\) \((0.075\cdot a)\) is the displacement of the center of the dominant magnetic exchange path which can either be \(J_2\) or \(J_3\) along the \(a\)- and \(b\)-axis, with respect to the screw-axis. The value of \(F_{anis}(q_{h,k,l})\) is the anisotropic magnetic form factor which takes into account the oxygen plaquette surrounding each magnetic \(\text{Cu}^{2+}\) ion and is given in Appendix B. The value of \(S_d(q,\omega)\) is the DSF of a single linear dimerized chain given by:

\[
S_d(q_{h,k,l},\omega) = (1 - \cos(q_l \cdot d_{2c(3c)})) \cdot (1 + \frac{\alpha}{2} \cos(q_l \cdot D)) \times \delta(\omega - \omega(q_{h,k,l}))
\]  

\tag{9}

where \(d_{2c(3c)}\) is the projection of the dominant magnetic exchange path onto the \(c\)-axis which equals \(d_{2c(3c)}=2.118\text{Å}(1.944\text{Å}).\)

The \(S_{sd}(q_{h,k,l},\omega)\)(eq. (7)) derived for \textit{Model 2} suggests that there are two non-degenerate modes \(\omega(q_l)\) and \(\omega(q_{l+1})\) which disperse along the \(l\) direction and are shifted by one reciprocal lattice parameter with respect to each other as observed experimentally. However, it is uncertain which mode in the INS data should be assigned to the \(\omega(q_l)\) dispersion relation and which to \(\omega(q_{l+1})\).

To extract the values of \(J_{intra}\) and \(\alpha\), the energy scans at the dispersion minimum and the dispersion maximum of the same mode were analyzed using both \(\omega(q_l)\) and \(\omega(q_{l+1})\) giving two sets of the solutions.

Figure 6 shows the constant wavevector scans collected at the dispersion minimum \((6,0,1)\) and maximum \((6,0,2)\) of the most intense mode along the \((6,0,l)\) direction. The white arrows on Fig. 4(a) indicate the wavevectors of measurements.

The dashed-dotted blue line in Fig. 6 gives the best fit of the INS data achieved using \(\omega(q_l)\) convolved with the resolution function. The instrumental resolution function calculated by the RESCAL software using the experimental settings of these particular measurements was convolved with the fitted function. The fitted parameters were \(J_{intra}=40.92\pm0.02\text{meV}\) and \(\alpha_{FM}=-0.2925\pm0.0025\) suggesting that in \(\text{BaCu}_2\text{V}_2\text{O}_8\) the AFM dimers are
weakly coupled by ferromagnetic (FM) interactions along the c-axis, we call this the AFM-FM solution. The solid red line through the data on the same figure presents the result using $\omega(q_{1})$. The parameters were found to be $J_{\text{intra}}=40.75\pm0.02$ meV and $\alpha_{\text{AFM}}=0.2250\pm0.0025$ suggesting that both exchange couplings in BaCu$_2$V$_2$O$_8$ are antiferromagnetic and this is called the AFM-AFM solution. Both solutions fit the experimental data equally well and neither solution is preferred. Both the AFM-FM ($J_{\text{intra}}=40.92$ meV, $\alpha_{\text{FM}}=-0.2925$) and AFM-AFM ($J_{\text{intra}}=40.75$ meV, $\alpha_{\text{FM}}=0.225$) solutions were compared with the results of fitting of the DC susceptibility, $\chi$. Only the AFM-FM solution is consistent with the values $J_{\text{intra}}=39.80$ meV and $\alpha_{\text{FM}}=0.248$ extracted from the dc-susceptibility data implying that the interdimer magnetic exchange coupling in BaCu$_2$V$_2$O$_8$ is ferromagnetic. Moreover, the simulations of $\chi_{H||c}$ and $\chi_{H\perp c}$ performed using the weakly-coupled dimer model keeping the interactions fixed for the AFM-FM solution, $J_{\text{intra}}=40.92$ meV and $\alpha_{\text{FM}}=-0.2925$ (green dashed line in Fig.2) reveal slightly better agreement with the experimental data than the simulation performed with the magnetic exchange constants fixed to the AFM-AFM solution, $J_{\text{intra}}=40.75$ meV and $\alpha_{\text{AFM}}=0.225$ (cyan dashed-dotted line in Fig.2). To provide further confirmation of the Hamiltonian of BaCu$_2$V$_2$O$_8$, the DSF $S_{sd}(q,\omega)$ was computed using equation (7) for both the AFM-FM and AFM-AFM solutions assuming that $J_3$ is the dominant magnetic exchange path.

Figure 6(e,f,g,h) and Figure 6(i,j,k,l) show the magnetic excitation spectrum of BaCu$_2$V$_2$O$_8$ computed along the (6,0,1), (5,0,1), (3,0,1) and (2,0,1) directions for AFM-FM and AFM-AFM solutions, respectively. The comparison of the simulations with the experimental data reveal that only the AFM-FM solution reproduces the intensity distribution correctly. Indeed, the DSF computed for the AFM-AFM Hamiltonian displays an opposite intensity distribution with respect that observed. This result was confirmed by the simulations performed along the $(h,0,4)$ and $(h,0,5)$ directions using $S_{sd}(q,\omega)$ for both AFM-FM and AFM-AFM models (Fig. 5).

Unfortunately, due to the small difference between $\Delta_3$ and $\Delta_2$, switching the dominant exchange path from $J_3$ to $J_2$ in $S_{sd}(q,\omega)$(eq. 7) does not change the DSF significantly, and therefore, cannot be used to determine which path gives the dominant antiferromagnetic interaction. To identify which magnetic exchange path $J_2$ or $J_3$ is responsible for the dominant dimer exchange interaction $J_{\text{intra}}$, the intensity distribution of the INS data were analyzed using the first moment sum rule:

$$\langle \omega \rangle_q = \hbar \int_{-\infty}^{\infty} \omega S(q,\omega)d\omega$$

$$\propto - \sum_{m=1,2,3} |F_{\text{anis}}(q_{h,k,l})|^2 \cdot J_m(S_{0} \cdot S_{d_m})(1 - \cos(q \cdot d_m))$$

(10)
Here, the contribution \( J_m(S_0 \cdot S_{d_m}) \) is proportional to the spin-spin correlation function between two spins coupled by the magnetic exchange interaction \( J_m \) and separated by the distance \( d_m \). Because \( J_m(S_0 \cdot S_{d_m}) \) is modulated by a particular periodicity \((1 - \cos(q \cdot d_m))\) there is a link between the first moment of the DSF and the relevant \( Cu^{2+}-Cu^{2+} \) distances within the crystal structure. Therefore, fitting the experimentally determined first moment to Equation (10) should highlight the dominant exchange paths although the absolute value of the exchange interaction \( J_m \) cannot be extracted.

Figure 7(a)-(b) show raw INS data measured along the \((0,0,l)\) and \((1,1,l)\) directions at the base temperature of \( T = 5K \). These particular directions were chosen because the maths is simplified for wavevector transfers with \( h = k \). Indeed, for a general wavevector transfer, the clockwise and anticlockwise screw-chains in the unit cell of BaCu\(_2\)V\(_2\)O\(_8\) produce different modulations for the same \( d_m \) because their projections on the wave-vector transfer are different. However, for the particular case of \( q_{h,l} \) both chains give the same modulation.

Figure 7(c)-(d) shows the background subtracted INS data along the \((0,0,l)\) and \((1,1,l)\) directions which were used to calculate the first moment of the dynamic structure factor plotted as a function of wavevector transfer in Figure 7(e)-(f), respectively. The extracted dependencies were analyzed using relation (10) counting contributions from each orientation of the \( J_3 \) and \( J_2 \) exchange paths, which are characterized by the distances \( d_3 = 3.007 \text{ Å} \) and \( d_2 = 2.87 \text{ Å} \), respectively. The solid blue lines through the data on Figure 7(c)-(d) show the best fit of the first moment of the DSF achieved for the values of \( J_3(S_0 \cdot S_{d_3}) = -984 \pm 192 \), \( J_2(S_0 \cdot S_{d_2}) = -255 \pm 122 \) along the \((0,0,l)\) direction, and \( J_3(S_0 \cdot S_{d_3}) = -769 \pm 177 \), \( J_2(S_0 \cdot S_{d_2}) = -500 \pm 163 \) along the \((1,1,l)\) direction. The negative sign indicates that the spin correlations agree with the proposed sign of the exchange constants and that this system is not frustrated.

For both the \((0,0,l)\) and \((1,1,l)\) directions the dominant contribution arises from correlations between the spins separated by the distance \( d_3 \) which corresponds to the \( J_3 \) super-exchange path. These results suggest that \( J_3 \) is the partner responsible for the dominant AFM \( J_{\text{intra}} \) while the weaker FM \( J_{\text{inter}} \) is realized within the double-plaquettes via the \( J_2 \) path. This result is in agreement with the Goodenough - Kanamori - Anderson rule. Indeed, the bridge angle of the Cu-O-Cu path of \( J_3 \) is 94° which is close to the crossover from AFM to FM interactions according to the Goodenough - Kanamori - Anderson rule and is certainly consistent with the weak FM interaction observed experimentally.

IV. SUMMARY AND CONCLUSIONS

In this work we comprehensively explored the magnetic properties of BaCu\(_2\)V\(_2\)O\(_8\) at base temperature using DC susceptibility and inelastic neutron scattering. The single crystal dc-susceptibility measurements reveal that BaCu\(_2\)V\(_2\)O\(_8\) has a non-magnetic ground state suggesting that this system is dimerized. The \( \chi_{H \parallel c} \) and \( \chi_{H \perp c} \) display isotropic behavior over the whole temperature range from 2K to 900K apart from the difference in their amplitude. This difference is attributed to the anisotropy of the g-factor due to the square-planar coordination of the magnetic Cu\(^{2+}\) ions. The extracted values of \( g_{H \parallel \text{plaq}} = 2.09 \pm 0.03 \) and \( g_{H \perp \text{plaq}} = 2.45 \pm 0.03 \) are in good agreement with values reported for other cuprates with planar geometry.

Both \( \chi_{H \parallel c} \) and \( \chi_{H \perp c} \) were fitted well by the weakly-coupled-dimer model over the whole temperature range. The extracted values of \( J_{\text{intra}} = 39.80 \text{meV} \pm 0.13 \text{meV} \) and \( J_{\text{inter}} = 9.87 \text{meV} \pm 2.64 \text{meV} \) imply that BaCu\(_2\)V\(_2\)O\(_8\) consists of AFM dimers which are weakly coupled ferromagnetically contradicting to all models in the literature which assumed that both interaction are AFM.

The single crystal INS measurements reveal that BaCu\(_2\)V\(_2\)O\(_8\) has a gapped magnetic excitation spectrum which disperses over the energy range from 35.37meV\(\pm0.05\text{meV} \) to 45.56meV\(\pm0.05\text{meV} \) along the \( \delta \)-direction but is completely dispersionless within the tetragonal plane implying a 1D-coupling of the dimers along the \( c \)-axis.

The observed magnetic excitation spectrum consists of two modes which are characterized by the same periodicity, size of the energy gap and bandwidth, but are shifted with respect to each other by half a period along the \( l \) direction. The dispersion of these modes was analyzed using the 5th order expansion of the one-magnon dispersion relation proposed for a dimerized chain. The extracted dimer periodicity equals half of the unit cell which exactly corresponds to the screw-chain model implying that magnetic exchange interactions in BaCu\(_2\)V\(_2\)O\(_8\) are realized via the Cu-O-V(II)-O-Cu (\( J_1 \)) and Cu-O-Cu (\( J_2 \)) paths and form a dimerized screw chain about the \( c \)-axis with two dimers per unit cell (model 2 in fig 1).

Two possible solutions of the Hamiltonian are given by fitting the dispersion of BaCu\(_2\)V\(_2\)O\(_8\) to the dimerized chain model. The first solution suggests that both \( J_{\text{intra}} \) and \( J_{\text{inter}} \) are antiferromagnetic and equal to \( J_{\text{intra}} = 40.75 \pm 0.02 \text{meV} \) and \( J_{\text{inter}} = 9.17 \pm 0.1 \text{meV} \). The second solution, in contrast, implies that only \( J_{\text{intra}} = 40.92 \pm 0.02 \text{meV} \) is antiferromagnetic while \( J_{\text{inter}} = -11.97 \pm 0.1 \text{meV} \) is ferromagnetic.

An analytical relation for the dynamic structure factor of the dimerized screw chain was derived to compute the magnetic excitation spectrum of BaCu\(_2\)V\(_2\)O\(_8\) at base temperature. The spectrum was computed for both solutions of the Hamiltonian. The comparison with the experimental data reveals that only the AFM-FM solution (\( J_{\text{intra}} = 40.92 \text{meV} \) and \( J_{\text{inter}} = -11.97 \text{meV} \)) reproduces the observed intensity distribution proving...
that the interdimer coupling is ferromagnetic. The Cu-O-V(II)-O-Cu super-super-exchange path was found to be responsible for the dominant AFM \( J_{\text{intra}} \) based on the analysis of the intensity distribution of the INS data using the first moment sum rule. A weak FM \( J_{\text{inter}} \) interaction lies within the copper-oxygen plaquettes which is consistent with the Goodenough-Kanamori-Anderson rules for super-exchange coupling.\(^{34-36}\)

To conclude, we show that \( \text{BaCu}_2\text{V}_2\text{O}_8 \) is a rare physical realization of an AFM-FM dimerized chain whose magnetic energy scale is much larger than those of previously reported AFM-AFM and AFM-FM dimerized chains.\(^{31,41,131}\) The Hamiltonian of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) is close to being ideally one-dimensional with negligible interchain coupling and was accurately solved so that the very strong antiferromagnetic and weaker ferromagnetic exchange paths were identified. Finally, the analytical relation for the dynamic structure factor of the unit cell of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) was derived and compared to the data to verify the extracted Hamiltonian.

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Appendix A: Dynamic structure factor for the dimerized screw chain.

This appendix describes an analytical relation for the dynamic structure factor (DSF) of the unit cell of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) where the copper-oxygen double-plaquettes, \( \text{Cu}_2\text{O}_6 \), form four dimerized screw-chains along the \( c \)-axis (Fig. 8(a,b,d) in the main text). Although the dynamic structure factor for the linear-dimerized chain is known\(^{3,2} \) the DSF for the screw-dimerized chain has not been derived.

The DSF of the dimerized screw-chain can be obtained using the DSF of the corresponding linear-dimerized chain\(^{2} \) which has the same Hamiltonian, using the relationship between the Bravais lattice of the linear chain and the non-Bravais lattice of the screw-chain where the arrangement of the magnetic sites is different.\(^{32} \) Figure 8(a) shows one of the four dimerized screw-chains per unit cell, formed by the \( J_2 \) (solid yellow line) and \( J_3 \) (solid double red line) magnetic exchange paths which are characterized by direct distances between the magnetic \( \text{Cu}^{2+} \) ions of \( d_2=2.87\,\text{Å} \) and \( d_3=3.077\,\text{Å} \), respectively. For clarity, only the magnetic \( \text{Cu}^{2+} \) ions and their coordinating oxygen ions are shown.

Figure 8(b)-(c) show the projections of this chain onto the \( (ac) \)- and \( (bc) \)-planes illustrating two possible scenarios where either the \( J_2 \) (Fig. 8(b)) or the \( J_3 \) (Fig. 8(c)) path is responsible for the dominant antiferromagnetic intradimer exchange coupling, \( J_{\text{intra}} \). The filled green diamonds give the centers of the dimers which are shifted perpendicular to the screw-axis by either \( \Delta_2=0.09\,\text{Å} \) or \( \Delta_3=0.075\,\text{Å} \) depending on whether the \( J_2 \) or \( J_3 \) path is responsible for the dominant intradimer coupling, respectively. The solid blue lines and filled blue squares give the projections of the dimers and their centers onto the screw-axis and are equal to \( d_{2c}=0.26\,\text{Å} \) if \( J_2=J_{\text{intra}} \) (Fig. 8(b)) or \( d_{3c}=0.2386\,\text{Å} \) if \( J_3=J_{\text{intra}} \) (Fig. 8(c)). The dashed gray lines are the projections of the weaker interdimer path onto the screw-axis and are given by \( d_{3c} \) on Fig. 8(b) and \( d_{2c} \) on Fig. 8(c), respectively. Thus, each dimerized screw-chain of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) (Fig. 8(a)) can be constructed from the dimerized linear-chain \( d_{2c}-d_{3c} \) by shifting the projections of the dimers perpendicular to the chain direction. The linear alternating-chain \( d_{2c}-d_{3c} \) lies along the screw-axis and has the same exchange constants \( J_2 \) and \( J_3 \) as the \( d_{2c}-d_{3c} \) dimerized screw-chains. This representation can be applied to each screw chain in the unit cell of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) (Fig. 8(e)-(f)). The dimerized linear-chains \( d_{2c}-d_{3c} \) form a Bravais lattice where the nodes are the centers of the dimers and edges are the screw-axes which are parallel to the \( c \)-axis. Assuming that both the linear \( d_{2c}-d_{3c} \) dimerized chains (Bravais lattice) and screw dimerized chains (non-Bravais lattice) have the same Hamiltonian, the DSF of the unit cell of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) \( S_{\text{sd}}(q,\omega) \) can be expressed using the relationship proposed between the DSF of Bravais and non-Bravais lattices\(^{32} \)

\[
S_{\text{sd}}(q_{h,k,l},\omega) = |F_{\text{anis}}(q_{h,k,l})|^2 \times \sum_{\mu=1}^{8} \left[ \sum_{m=1}^{8} \exp(-i \cdot q_{h,k,l} \cdot \rho_m) \exp(-i \cdot T_{\mu} \cdot r_m) \right] \cdot S_d(q_{h,k,l} + T_{\mu},\omega) \tag{A1}
\]

where the inner sum is taken over the magnetic sites \( \vec{r}_m \) which are the centers of the dimers in the dimerized linear-chains and each \( \vec{r}_m \) correspond to one of the eight magnetic sites \( \vec{R}_m \) (centers of the corresponding dimers in the screw chains) within the single unit cell of \( \text{BaCu}_2\text{V}_2\text{O}_8 \). The relation between \( \vec{r}_m \) and \( \vec{R}_m \) is given by the displacement vector \( \vec{\rho}_m \) so that \( \vec{R}_m = \vec{r}_m + \vec{\rho}_m \) where \( |\vec{\rho}_m| = \Delta_{2(3)} \) depending on which path \( J_2 \) or \( J_3 \) is responsible for \( J_{\text{intra}} \). The outer sum is taken over all reciprocal lattice vectors \( T_{\mu} \) of the non-Bravais unit cell of \( \text{BaCu}_2\text{V}_2\text{O}_8 \) within the first Brillouin zone of the Bravais unit cell. 

\( S_d(q_{h,k,l},\omega) \) is the DSF for the dimerized linear-chain.
FIG. 8. (a) A single screw-chain in the unit cell of BaCu$_2$V$_2$O$_8$. For clarity, only the Cu$^{2+}$ magnetic ions (filled blue spheres) and oxygen atoms (filled gray spheres) within the plaquettes are shown. The dotted line indicates the screw-axis. The solid yellow and double red lines give the magnetic exchange paths $J_2$ and $J_3$, respectively. Projection of the dimerized screw-chain of BaCu$_2$V$_2$O$_8$ on (b) the ac-plane illustrating the scenario where $J_2 = J_{\text{intra}}$ and on (c) the bc-plane illustrating the scenario where $J_3 = J_{\text{intra}}$. The filled green diamonds give the centers of the dimers in the dimerized screw-chain of BaCu$_2$V$_2$O$_8$. $d_{2c}$ and $d_{3c}$ are the projections of the $d_2$ and $d_3$ paths on the screw-axis. The solid blue lines and filled blue squares give the projections of the dominant path (dimer) and its center on the screw-axis. The Bravais unit cell of the linear chains constructed from the real unit cell of BaCu$_2$V$_2$O$_8$ where (e) $J_2 = J_{\text{intra}}$ and (f) $J_3 = J_{\text{intra}}$. The method for this construction is described in the text of Appendix A.

which is given by:\n
\[ S_d(q_{h,k,l},\omega) = (1 - \cos(q_l \cdot d_{2c}(3c))) \cdot \left(1 + \frac{\alpha}{2}\cos(q_l \cdot D)\right) \times \delta(\omega - \omega(q_{h,k,l})) \] \n
(A2)

where $q_{h,k,l}$ is the wave-vector transfer and $\omega$ is the frequency. The energy dispersion relation $\omega(q_{h,k,l})$ for the one-dimensional dimerized chain is given in the main text (eq. 5) and was originally derived in Ref[2]. $D$ is the dimer periodicity which is $D = \frac{c}{2}$ for BaCu$_2$V$_2$O$_8$.

There are eight of each of $T_{\mu}$, $r_m$ and $\rho_m$ which are summarized in Table 1 for the two scenarios, $J_{\text{intra}} = J_2$ or $J_{\text{intra}} = J_3$. The substitution of the corresponding values into equation A1 reveals that only $T_1$, $T_6$, $T_7$ and $T_8$ give non-zero contributions for both cases of $J_{\text{intra}}$.

For the case where the $J_3$ path is responsible for the dominant intradimer coupling $J_{\text{intra}}$, the DSF of BaCu$_2$V$_2$O$_8$...
For the scenario where the J\(\psi\) is given by:

\[
\psi_1 = q_h \cdot \Delta_3; \quad \psi_2 = q_k \cdot \Delta_3
\]

For the scenario where the J\(2\) path is responsible for the dominant J\(\text{intra}\), the DSF is given by:

\[
S_{sd}(q_h,k,l,\omega) = |F_{\text{anis}}(q_h,k,l)|^2 \times \left(4 \cdot \cos(\psi_2) + 4 \cdot \cos(\psi_1)\right)^2 S_d(q_h,k,l,\omega) + |4 \cdot \cos(\psi_2) - 4 \cdot \cos(\psi_1)|^2 \cdot S_d(q_h+1,k+1,l,\omega) + 4 \cdot \sin(\psi_1) + 4 \cdot \sin(\psi_2)\]  

(\(q_{h}+1, k+1, l, \omega\))

where \(\psi_1\) and \(\psi_2\) are given by

\[
\psi_1 = q_h \cdot \Delta_2; \quad \psi_2 = q_k \cdot \Delta_2
\]

Because the energy dispersion for the linear dimerized chain depends only on the component of the wave-vector transfer \(q\) which is along the chain direction, the modes \(S_d(q_h,k,l,\omega)\) and \(S_d(q_h+1,k+1,l,\omega)\) which have the same \(q\) are degenerate as are \(S_d(q_h,k,l,\omega)\) and \(S_d(q_h+1,k+1,l,\omega)\).

Thus, both equations A3 and A5 can be simplified and written in the general form

\[
S_{sd}(q_h,k,l,\omega) = |F_{\text{anis}}(q_h,k,l)|^2 \times \left(\cos(\psi_1)^2 + \cos(\psi_2)^2\right) \cdot S_d(q_h,k,l,\omega) + (\sin(\psi_1)^2 + \sin(\psi_2)^2) \cdot S_d(q_h+1,k,l,\omega)
\]

(\(q_{h}+1, k, l, \omega\))

where

\[
\psi_1 = q_h \cdot \Delta_3; \quad \psi_2 = q_k \cdot \Delta_3 \quad \text{for} \quad J_2=J_{\text{intra}}
\]

\[
\psi_1 = q_h \cdot \Delta_3; \quad \psi_2 = q_k \cdot \Delta_3 \quad \text{for} \quad J_3=J_{\text{intra}}
\]

Finally, \(F_{\text{anis}}(q_h,k,l)\) is the anisotropic magnetic form factor which takes into account the planar surrounding of the magnetic copper ions and is discussed below in Appendix B.

Appendix B: Anisotropic magnetic form factor for the copper ions in the planar coordination.

In BaCu\textsubscript{2}V\textsubscript{2}O\textsubscript{8} the magnetic Cu\textsuperscript{2+} ions are surrounded by the O\textsuperscript{2–} anions forming planar copper-oxygen Cu\textsubscript{2}O\textsubscript{6} double-plaquettes. The magnetically active 3d\textsubscript{z\textsuperscript{2}–y\textsuperscript{2}} electronic orbitals of Cu\textsuperscript{2+} lie within these double-plaquettes forming strong covalent d-p bonds with the O 2p orbitals and the z-axis is perpendicular to the plaquettes. This leads to an anisotropic magnetic form factor of the magnetic Cu\textsuperscript{2+} ions within the copper-oxygen double-plaquettes which is given by:

\[
F_{\text{plaq}}(q) = \langle j_0 \rangle - 5 \theta (1 - 3 \cdot \cos^2(\theta)) \cdot \langle j_2 \rangle + \frac{9}{56} (1 - 10 \cos^2(\theta)) + \frac{35}{4} \cos^4(\theta) \cdot \langle j_4 \rangle
\]

(\(B1\))

Here, \(\langle j_0 \rangle, \langle j_2 \rangle\) and \(\langle j_4 \rangle\) are spherical Bessel functions.
and $\theta$ is the angle between the wavevector transfer and the normal to the plaquettes.\textsuperscript{37}

In BaCu$_2$V$_2$O$_8$ the Cu$_2$O$_6$ double-plaquettes have two different orientations and contain either the $a$-c or $b$-c crystal axes within the plaquette plane. Therefore, the magnetic form factor for the Cu$^{2+}$ ions in BaCu$_2$V$_2$O$_8$ should be averaged over the two orientations of the plaquettes and is given by:

$$|F_{\text{anis}}(q_h,k,l)|^2 = \left(\frac{|F_{\text{plaq}}^1(q)|^2}{2} + \frac{|F_{\text{plaq}}^2(q)|^2}{2}\right)$$  \hspace{1cm} (B2)

Here, the values of $F_{\text{plaq}}^1(q)$ and $F_{\text{plaq}}^2(q)$ describe the magnetic form factors of the Cu$_2$O$_6$ double-plaquettes oriented perpendicular to the $a$- and $b$-crystal axes, respectively and are given by equation (B1), with $\cos(\theta) = \frac{q_a}{|q|}$ and $\cos(\theta) = \frac{q_b}{|q|}$, respectively. Note, that for wavevectors $q_{h,k,l}$ with $h=k$ $F_{\text{plaq}}^1(q)$ and $F_{\text{plaq}}^2(q)$ are the same which simplifies the expression for the anisotropic magnetic form factor $F_{\text{anis}}(q_{h,k,l})$.


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