

RECONSTRUCTION OF THE TRANSVERSE ELECTRON BEAM PROFILE USING AN INTERFEROMETRIC BEAM SIZE MONITOR*

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Abstract

The transverse size of the electron beam in a storage ring can be measured using the synchrotron radiation of a bending magnet. Due to the diffraction limit, many facilities exploit beam size monitors in the X-ray regime. On the other hand, the visible part of the emitted radiation delivers spatial information via an interference pattern after passing through a double slit. Assuming a Gaussian beam distribution the size of the beam can be easily obtained with an analytical formula. If this assumption is not fulfilled, the calculated beam shape will differ from the real distribution. This can appear for instance in the case of exotic beam optics settings or complicated filling patterns, that are widely used in modern storage-ring-based light sources. In this paper, the idea to reconstruct the electron beam distribution by determining the absolute visibility and its phase with a spectral-resolved set-up is introduced.

INTRODUCTION

Accelerator beam physics progresses recently towards a small beam emittance from a few nanometer-radians to in the order of tens of picometer-radians, resulting in a beam size in the order of fewer than ten micrometers, requiring appropriate beam diagnostics to determine transverse beam sizes with high precision. One already well-known method is visible interferometry which quantifies the size of the source by analyzing spatial coherency of the diffraction pattern using the visible part of synchrotron radiation [1, 2]. For this method it is necessary to assume a reference distribution of the beam to analyze coherency. The Gaussian distribution is widely adopted. However, it can lead to misreading of the beam size for extreme conditions. To overcome the limitation, there were some efforts to reconstruct the size and shape of the beam through the investigation of the amplitude of visibility and phase changes by varying the slit separation [1, 3]. The beam profile can be reconstructed in previous approaches, but this is not suitable for a single measurement since it requires multiple measurements of visibility and phase values with different slit separations. Here, we propose a novel approach that enables the detection of visibilities and phases in a single measurement by converting the spectral distribution of synchrotron radiation to space using a prism. In the following, beam profile reconstruction with the spectral-resolved set-up is introduced in detail.

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THEORY

In order to comprehend the concept of the beam profile reconstruction using the spectral resolved set-up, it is required to briefly review the theory of the interferometric beam size monitor utilizing synchrotron radiation, which was first proposed by T. Mitsuhashi [4]. The interference pattern for Young's double-slit interferometer with a finite source size can be described as:

$$I(x) = (I_1 + I_2) \operatorname{sinc}^2\left(\frac{a}{\lambda f}x\right) \left[1 + V \cos\left(\frac{2\pi d}{\lambda f}x + \psi\right)\right], \quad (1)$$

where x is the position on the detector, a is the full width of a single slit, d is the distance between slits, f is the distance between the focusing lens and the detector screen, λ is the wavelength, I_1 and I_2 are intensities of the light at both slits, respectively, ψ is the phase, and V is the absolute amplitude of coherency which is called as visibility. The visibility of the interferogram depends not only on the spatial coherency between photons from two slits but also on the intensity imbalance ratio. It can be roughly represented by the ratio of the difference and sum between the maximum intensity value I_{\max} and intensity in the first local minima I_{\min} as

$$V \approx \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} |\gamma|, \quad (2)$$

where γ is the complex degree of spatial coherence. This physical quantity is defined by the van Cittert-Zernike theorem as the Fourier transform of the intensity distribution of the source:

$$\gamma(d, \lambda, L) = \int f(x) \exp\left(-i \frac{2\pi d}{\lambda L} x\right) dx, \quad (3)$$

with L - the distance from the source to the double slit and $f(x)$ - the intensity distribution of the source. Thereby the complex degree of spatial coherence is a function of the amplitude visibility V and phase ψ . In the case of equal intensity of incident light at both slits, $I_1 = I_2$, the visibility becomes the absolute value of the complex degree of spatial coherence, so it can be written as

$$\gamma(d, \lambda, L) = V(d, \lambda, L) e^{i\psi(d, \lambda, L)}, \quad (4)$$

where the phase is given by

$$\psi(d, \lambda, L) = \arctan \frac{\operatorname{Re}(\gamma(d, \lambda, L))}{\operatorname{Im}(\gamma(d, \lambda, L))}. \quad (5)$$

Therefore the initial profile of the source point can be reconstructed by inverse Fourier transformation of the complex

degree of spatial coherence. In general, it is possible to measure visibility and phase by varying either the slit separation, wavelength, or distance from the source point to the double slit.

The distance from the source point to the slit is not physically suitable because it can not scan sufficient visibility regions for the inverse Fourier transformation. The previous studies showed that adjusting slit separation manually can easily obtain enough data for the reconstruction of the beam distribution. Here we propose the reconstruction of the beam distribution by a spectral resolved measurement using only an additional prism or diffraction grating which is necessary to separate all the wavelengths in space at a detector. The interference patterns for all the wavelengths can be recorded in a single picture (see Fig. 1). In the following section the method of the spectral resolved measurements will be investigated.

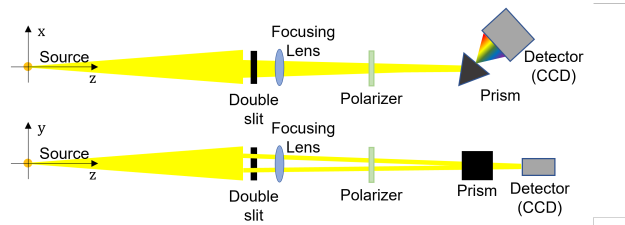


Figure 1: Schematic layout of spectral resolved measurements.

SIMULATIONS

For the reconstruction method based on Fig. 1 it is essential to understand the visibility behaviour since visibility changes as a function of wavelength. The behaviour is calculated for several beam intensity profiles which are differently disturbed Gaussian distributions. The three different beam distributions are mainly considered that are shown in Fig. 2: *ideal case*: perfectly Gaussian distributed beam (blue curve); *symmetric overlap*: two Gaussian distributed profiles with different sizes and amplitudes are located at the center. One profile has a smaller amplitude with a larger beam size than the other (orange curve). This case is expected to be close to the beam distribution when beam halo exists in a storage ring; *asymmetric overlap*: beam profile with overlap with a secondary electron distribution of a similar size, but shifted relative to the main beam (green curve).

The visibility curves as a function of wavelength are simulated for the described cases. The result is shown in Fig. 3. All three curves show the same asymptotic behaviour: start from zero and get saturated around one. However, in the middle, they have different shapes depending on the corresponding initial distribution. It is important at this point to understand which part of the curve can be investigated with the real measurement. Experimentally, the most straightforward way is to use the visible part of the light. Figure 3 indicates that the visible part of the spectrum has a noticeable discrepancy for the plotted example curves.

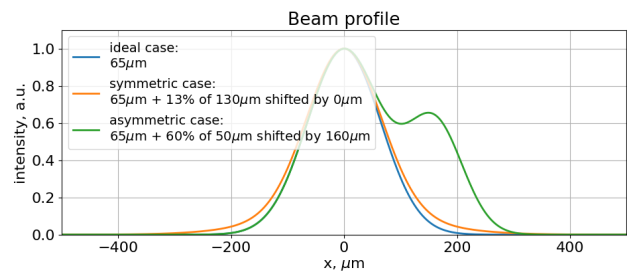


Figure 2: Three conceivable beam profiles which are used in simulations (given % refers to the high of the main peak).

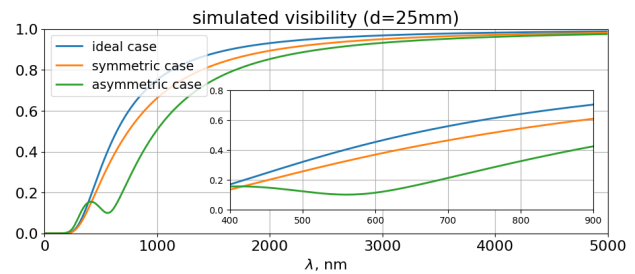


Figure 3: Simulated visibility as a function of wavelength for the three profiles shown in Fig. 2.

Figure 3 also shows that the strongest change of visibility happens over a relative short frequency range. Therefore the slit separation has to be optimized to shift this region into visible light. As shown in Fig. 4, different slit separations in the set-up lead to different results. Following the position of the local minima in all the plotted curves, it is clear that depending on the chosen slit separation different parts of the visibility curve are reproduced.

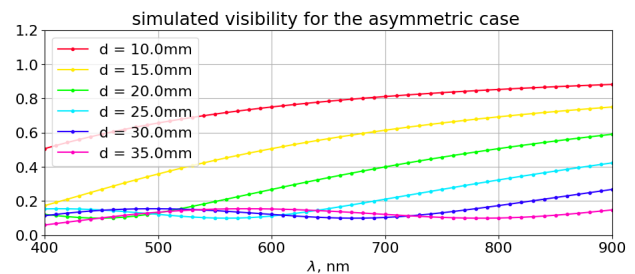


Figure 4: Visibility in the visible range for different slit separations. Simulations for the asymmetric distribution.

Important to note is, that transitional phases for the distributions with symmetric and asymmetric overlaps are different: in the second case there exist clear local minima, though in the first case, the transition is just a slow change from one visibility curve to the other. In spite of that, the finding of this region enables better description and extrapolation of the presented curve.

After visibility and phase values in the visible range are determined from the experiment, it is necessary to extrapolate the measured function in the whole space (from $-\infty$ to $+\infty$) to be able to reconstruct the initial profile. Therefore,

suitable functions must be found that can be fitted to the measured data points and deliver appropriate extrapolation results. Figure 5 shows the corresponding phase curves for three cases shown in Fig. 3. It can be seen that phases for the first two cases remain zero, which matches theoretical expectations for a symmetric distribution. Contrary to these curves, the phase of the last case is changing strongly. This behaviour has to be understood first to be able to extrapolate this function based only on the information from the visible range and then to complete the reconstruction process. Therefore, in the following section, only the symmetric case is analyzed.

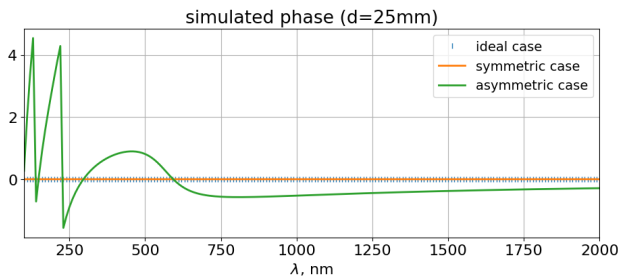


Figure 5: Simulated phase for the initial profiles from Fig. 2.

When a main distribution is overlapped with the wider one at the same position, the measured visibility curves in the visible part still look very similar to a single Gaussian beam. The wider profile causes a distortion of the visibility curve. Whereby the difference between the two is difficult to see in the visible range. However, by consideration of the larger wavelength range, it becomes obvious (see Fig. 6) since the visibility changes happen at longer wavelengths as the beam size increases.

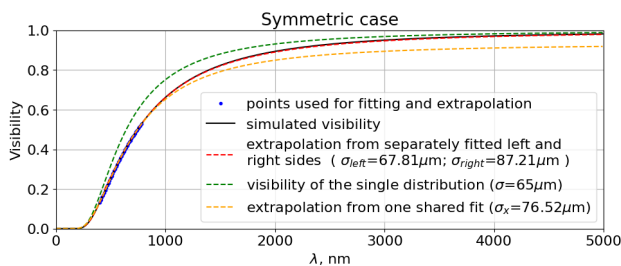


Figure 6: Comparison of the visibility determined by different fitting methods. Simulations for the symmetric distribution.

At this moment, assuming a symmetric distribution, the initial distribution can be retrieved by the extrapolation of visibility measured in the visible region. In Fig. 7, the results of the reconstruction are shown and compared with the initial profile. It evidences that the quality of the reconstruction relies on the method of extrapolation. Especially, the results suggest that preciser retrieving can be achieved by two distinct fittings at shorter and longer wavelengths.

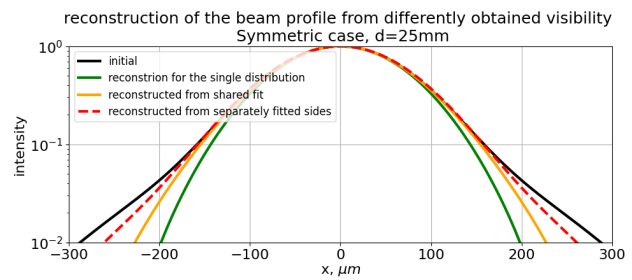


Figure 7: Comparison of the reconstructed beam profiles obtained from different extrapolation methods.

POSSIBILITIES AND LIMITATIONS

Following the described process of the beam profile reconstruction, some limitations of this kind of analysis can already be clearly seen. First of all, since it is not possible experimentally to measure the visibility and phase values over the entire spectral range, extrapolation is strongly needed and it influences the results of the obtained profile. Therefore, it might be difficult to reconstruct some profiles, for instance the asymmetric ones. Moreover, the fitting and extrapolation already approximate the initial profile to certain functions which satisfy the condition. This might be not always practical. To get phase values various phase retrieval algorithms might be helpful (like in Refs. [3, 5]).

However, for more predictable cases, the use of this method can save computing time enormously for the diagnostic of the beam profile. Also, the reconstruction process can be applied not only for the integrated beam but also for individual bunches. These spectral resolved visibility measurements can deliver enough information for the evaluation of the quality of the profile so that it can be seen how strongly the real profile deviates from the expected one.

Dependency of the obtained visibility and phase on the used slit separation can be considered as both: a limitation and a possibility. On one hand, using one chosen double slit, i.e. chosen slit separation, only a small part of the visibility and phase curves can be received. On the other hand, usage of a set of double slits (for instance having three different slit separations) can deliver information about visibility and phase curves in a much larger range.

CONCLUSION

To surpass the limitation of the interferometric beam size monitor which measures a beam size with the assumption of a certain distribution, reconstruction of beam distribution can be used. Possibility to reconstruct beam distribution using a spectral resolved set-up for interferometric measurement has been discussed and demonstrated through computer simulations on three different scenarios. The result shows that the initial distribution is well retrieved for simple symmetric distributions. It is important to note, however, that the introduced analysis has limitations that must be considered before applying the method to the real case.

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