The following was adapted from the Wikipedia entry "Superconducting radio frequency" (as of Aug. 2013)

Superconducting radio frequency

Figure 1: (left) A 1.5-GHz superconducting single-cell RF cavity ("CEBAF shape"). (right) A 9-cell 1.3-GHz multicell cavity designed for the international linear collider. Both cavities are made of niobium.

Superconducting radio frequency (SRF) science and technology involves the application of electrical superconductors to radio frequency devices. The ultra-low AC electrical resistivity of a superconducting materials allows an RF resonator to obtain an extremely high quality factor (Q) which is a measure of the number of oscillations needed to dissipate the energy stored in the electromagnetic field. For example, it is commonplace for a 1.3 GHz niobium SRF resonant cavity at 1.8 Kelvin to obtain a quality factor of $Q = 5\times10^{10}$. These properties can be exploited for a variety of applications, including the construction of high-performance particle accelerator structures.

The amount of loss in an SRF resonant cavity is so minute that it is often explained with the following comparison: Galileo Galilei (1564–1642) was one of the first investigators of pendulous motion, a simple form of a mechanical resonance. Had Galileo experimented with a 1 Hz resonator (pendulum) with a quality factor Q typical of today's SRF cavities and started it swinging in the early 17th century, that pendulum would still be swinging today with about half of its original amplitude.

The most common application of superconducting RF is in particle accelerators. Accelerators typically use resonant RF cavities formed from good electrical conductors in which electromagnetic fields are excited resonantly by an external power source to accelerate charged particles. Originally, copper was the material of choice because it is maliable, is strong and has a high conductivity. However, at the electric field levels of interest in particle accelerators (millions of volts per meter, or MV/m) copper will still dissipate a prohibitively large amount of power when operated continuously (CW). Hence many modern accelerator applications now use cavities made of, or coated with, superconducting materials.

Electromagnetic fields are excited in the cavity by coupling in an RF source with an antenna. When the RF frequency fed by the antenna is the same as that of a cavity mode, resonant fields as shown in Figure 2 build to high amplitudes. Charged particles passing through apertures in the cavity are then accelerated by the electric fields provided the timing between the oscillating field and the particles is properly adjusted. The resonant frequency driven in SRF cavities typically ranges from 200 MHz to 3 GHz, depending on the particle species to be accelerated.

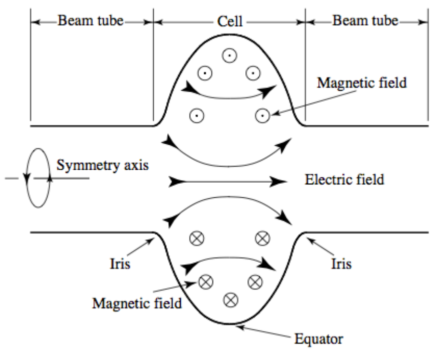

Figure 2: Cross section through a CEBAF cavity (as shown in Figure 1). Electrons enter on axis through the beam tube to be accelerated by the oscillating longitudinal electric field.

The most common fabrication technology for such SRF cavities is to form thin walled (1–3 mm) shell components from high purity niobium sheets by stamping (deep drawing). These shell components are then welded together in an electron-beam welder to form cavities. Several such finished products are pictured in Fig. 3.

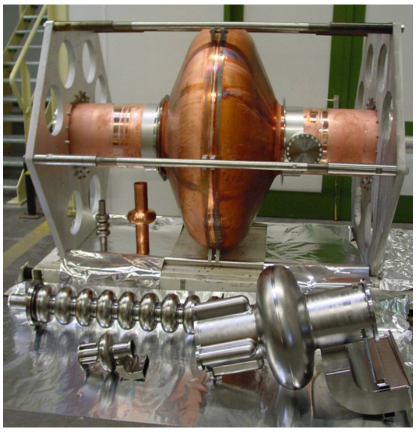

Figure 3: A collection of SRF cavities developed at Cornell University with frequencies spanning 200 MHz to 3 GHz (smallest cavity standing upright). Note that the largest cavity is made if copper that was coated with superconducting niobium on the inside.

A simplified diagram of the key elements of an SRF cavity setup is shown below. The cavity is immersed in a saturated liquid helium bath. Pumping removes helium vapor boil-off and controls the bath temperature. The helium vessel is often pumped to a pressure below helium's superfluid lambda point to take advantage of the superfluid's high thermal conductivity properties. Because superfluid has very high thermal conductivity, it makes an excellent coolant. In addition, superfluids boil only at free surfaces, preventing the formation of bubbles on the surface of the cavity, which would cause mechanical perturbations.

An antenna is needed in the setup to couple RF power to the cavity fields and, in turn, any passing particle beam. The cold portions of the setup need to be extremely well insulated, which is best accomplished by a vacuum vessel surrounding the helium vessel and all ancillary cold components. The full SRF cavity containment system, including the vacuum vessel and many details not discussed here, is a cryomodule as shown in Fig. 4.

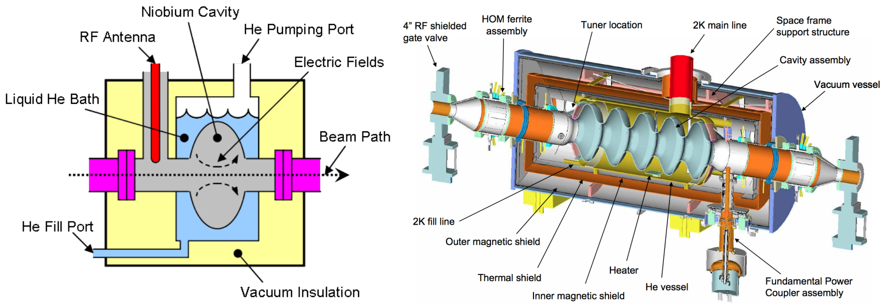

Figure 4: (left) Schematic of an SRF cavity in a helium bath with RF coupling and a passing particle beam. (right): Cross section of a crymodule with SRF cavity developed by Brookhaven National Laboratory for an Energy Recovery LINAC. Liquid helium fills the space between the cavity and the (yellow) helium tank.

Entry into superconducting RF technology can incur more complexity, expense, and time than normal-conducting RF cavity strategies. SRF cavities must first undergo a harsh chemical treatment to remove material at the surface that was damaged during the production. Any particles that remain on the surface (even micron-sized dust particles) can severely compromize the cavity performance. Hence a low-particulate cleanroom for high-pressure water rinsing and assembly of components is required. And complex engineering for the cryomodule vessel and cryogenics is required to satisfy the taxing vacuum and temperature specifications. A vexing aspect of SRF is the as-yet elusive ability to consistently produce high Q cavities in high volume production, which would be required for a large linear collider such as the International Linear Collider (ILC). Nevertheless, for many applications the capabilities of SRF cavities provide the only solution for a host of demanding accelerator performance requirements.

Several extensive treatments of SRF physics and technology are available, many of them free of charge and online. There are the proceedings of CERN accelerator schools [1-3], a scientific paper giving a thorough presentation of the many aspects of an SRF cavity to be used in the International Linear Collider [4], bi-annual International Conferences on RF Superconductivity held at varying global locations in odd numbered years [5] and tutorials presented at the conferences. More information can also be found in the books on SRF [6, 7].

Why use superconducting RF systems?

A large variety of RF cavities are utilized in particle accelerators. Historically they have been made of copper, a good electrical conductor, and operated near room temperature with water cooling. The water cooling is necessary to remove the heat generated by the electrical loss in the cavity. In the past two decades, though, there has been a growing number of accelerator facilities and new proposals for which superconducting cavities are the enabling technology.

The motivation for using superconductors in RF cavities is *not* (only) to achieve a net power savings. While superconductors can have zero DC resistivity, their AC resistivity does not vanish. Still, materials such as Nb used for cavities dissipate up to one million times less power than copper. However, this power is dissipated at very low temperatures, typically in a liquid helium bath between 1.6 K to 4.5 K. The refrigeration power to maintain the cryogenic bath at low temperature in the presence of the RF power dissipation is dictated by the Carnot and technical efficiency of the cryoplant. As a result the power savings with respect to copper will “only” be about a factor of 100 to 1000. While this power savings itself is attractive, its low value has a number of indirect advantages that often provides a much stronger motivation to use SRF rather than normal-conducting cavities.

- **High duty cycle or CW operation**. SRF cavities allow operation at high accelerating field at high duty cycle, or even continuous wave (CW), in such regimes that a copper cavity's electrical loss will *melt* the copper, even with robust water cooling.
- **Low beam impedance**. The low losses in an SRF cavity allows one the freedom to optimize its geometry to minimally disrupt the particle beam. For example, superconducting cavities naturally favor low frequencies and can have large beampipe apertures while still maintaining a high accelerating field. In contrast, normal-conducting cavities need small beam apertures to concentrate the electric field and high frequencies to minimze the power losses due to wall currents. However, this situation is deleterious to a particle beam due to their spawning of electromagnetic wakefields (“higher-order modes”), which are quantified by the cavity parameters termed “beam impedance” and “loss parameter.” If these wakefields are too severe they can cause beam instabilities that limit the amount of current that can be accelerated.
- **Nearly all RF power goes to the beam**. The RF source driving the cavity need only provide the RF power that is absorbed by the particle beam being accelerated, since the RF power dissipated in the SRF cavity walls is negligible. This is in contrast to normal-conducting cavities where the wall power loss can easily equal or exceed the beam-power consumption. The RF power budget is important since the RF source technologies, such as a klystron, inductive output tube (IOT), or solid state amplifier, have costs that increase significantly with increasing power.

Material used for SRF cavities

Niobium, cooled to temperatures below 4.3 K, is commonly used for SRF systems and represents the state-of-the-art. It has the highest superconducting critical temperature (T_c = 9.2 K) of all the elements and mechanical properties that are well suited for SRF cavity production. In the past, Pb has also been used. However, since it is mechanically soft it must be coated on a substrate such as copper or stainless steel to give the cavity sufficient mechanical strength.

Niobium remains superconducting up to a critical magnetic field of about 200 mT. If the magnetic field component of the RF field exceeds this value anywhere along the cavity wall, the niobium goes normal conduction, i.e., the cavity quenches.

Other (compound) materials such as Nb_3Sn are being investigated that may have a higher superconducting critical temperatures or critical magnetic field. They may therefore be operated at higher bath temperatures, with higher refrigeration efficiency or at higher field strength. But there are other issues that would have to be considered with a higher bath temperature, though, such as the absence of superfluidity that is presently exploited with liquid helium that would not be present with, e.g., liquid nitrogen.

High T_c superconductors have also been investigated but were found to be unsuitable for RF cavity applications. Shortcomings of these materials arise due to their underlying physics and anisotropy, as well as their bulk mechanical properties not being amenable to fabricating accelerator cavities.

Physics of (S)RF cavities

The physics of SRF can be complex and lengthy and exceeds the scope of this introduction. A few simple approximations derived from the complex theories, though, can serve to provide some of the important parameters of SRF cavities.

RF cavities in general

RF cavities in general (both normal conducting and superconducting) in their simplest form consist of hollow metal cylinders terminated by condicting end walls. If microwaves are injected via a coupling loop, only distinct field configurations are possible that satisfy the boundary conditions for electric and magnetic fields at the conducting walls. These are the cavity's resonant modes, each operating at a different frequency. The lowest mode for a so-called pillbox cavity as in Fig. 5 is the TM_{010} mode, that has a longitudinal electric field which varies as the 0^{th}-oder Bessel function (J_0) as one moves from the cavity axis towards the outside wall. No electric field exists at the outer wall of the cavity. This mode can accelerate charges along the cavity axis. In addition, there is an azimuthal magnetic field. Other, high-frequency modes will have a very different field configuration. Usually one tries to excite only the TM_{010} mode with an external microwave source by tuning the generator frequency to the mode's frequency.

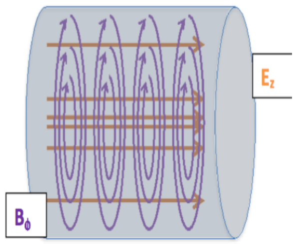

Figure 5: A simple pillbox cavity in which the TM_{010} mode has been excited.

For a given electric field strength in the cavity one can calculate how much energy a particle will receive. It is simply given by the integral of the longitudinal electric field through the cavity, at the particle position as it flies through the cavity, taking into account the harmonic time dependence of the field. If the speed of the particle does not change then

$$V_{\text{acc}} = q\int_{-\frac{L}{2}}^{\frac{L}{2}} E\left({}^{z}/_{\beta c}\right) \exp\left[i\omega\left({}^{z}/_{\beta c}\right) + i\phi\right] dz$$

where L is the length of the cavity, ω is the resonant frequency and β is the speed of the particle divided by c (we assume this to be constant in the cavity) and φ is the phase at which the particle traverses the cavity.

To maintain the RF field in the cavity, currents flow in the cavity walls that then dissipate some power. To characterize the "efficiency" of a cavity mode, one often quotes the quality factor Q_0 as a figure of merit. It is the number of RF cycles the cavity requires to dissipate the energy stored in its electromagnetic field. By convention one multiplies this by 2π:

$$Q_0 = 2\pi \frac{\text{Energy stored}}{\text{Energy dissipated in an RF cycle}} = \frac{\omega U}{P_{\text{diss}}},$$

where U is the energy stored, and P_{diss} is the power dissipated in the walls to maintain the energy U. The stored energy in the cavity is given by the integral of field energy density over its volume,

$$U = \frac{\mu_0}{2}\iiint |H|^2 dV.$$

Here H is the magnetic field in the cavity and μ_0 is the permeability of free space.

The dissipated power is determined by the surface currents flowing in the cavity wall. These are proportional to the local (AC) magnetic field of the mode's field distribution so that the wall losses/area can be written as

$$\frac{P}{A} = \frac{R_s}{2}\left|\vec{H}\right|^2.$$

The material dependent proportionality constant R_s is the surface resistance which will be discussed below. Thus the total power dissipation in the cavitiy is

$$P_{\text{diss}} = \frac{R_s}{2}\iint\left|\vec{H}\right|^2 ds.$$

The integrals of the electromagnetic field in the above expressions are generally not solved analytically since the boundaries of real cavities rarely lie along axes of common coordinate systems. Instead, the calculations are performed by any of a variety of computer programs that solve for the fields for non-simple cavity shapes, and then numerically integrate the above expressions.

Given the expressions for the stored energy and the power dissipation, one thus obtains for the quality factor

$$Q_0 = \frac{\omega\mu_0 \iiint |H|^2 dV}{R_s \iint\left|\vec{H}\right|^2 ds} = \frac{G}{R_s}$$

where the geometry factor is

$$G = \frac{\omega\mu_0 \iiint |H|^2 dV}{\iint\left|\vec{H}\right|^2 ds}.$$

If one considers the scaling of all cavity dimensions by a factor κ, one can convince oneself that the volume integral scales as κ^3, the surface integral as κ^2 and the frequency as κ^{-1}. Hence the geometry factor is only dependent on the cavity geometry but not its size (frequency). The geometry factor is quoted for cavity designs to allow comparison with other designs, independent of material characteristics and operating frequency, since wall loss for SRF cavities can vary substantially depending on material preparation, cryogenic bath temperature, electromagnetic field level, and other highly variable parameters.

As an example of the above parameters, a typical 9-cell SRF cavity for the International Linear Collider[4] (a.k.a. "TESLA cavity") would have G=270 Ω and R_s= 10 nΩ, giving Q_o=2.7×10^{10}. If this cavity were to operate at 15 MV/m it would dissipate only about 8.6 W of power! This value is very low, but one must bear in mind that the power is dissipated in liquid helium which must be cooled by a complex cryoplant. Its efficiency will only be of the order of 0.1% so that the wall-plug power to cool the cavity is of order 8 kW. For SRF cavities the power dissipation thus remains a non-neglibile consideration to minimize the operating cost and investment cost, as well as complexity of the cryogenic plant.

For a complete picture one has to determine how much acceleration a mode is able to supply for a given amount of power dissipation. Ideally this should be maximized. Hence one quotes the shunt impedance R_a of a mode, which is defined as

$$R_a = \frac{V_{\text{acc}}^2}{P_{\text{diss}}}.$$

Using the same arguments as above, one can also show that the shunt impedence scales as a geometric constant divided by R_s.

The vital parameter for SRF cavities in the above equations is the surface resistance R_s, and this is where the complex physics of superconductivity comes into play. For normal-conducting copper cavities operating near room temperature, R_s is simply determined by the bulk electrical conductivity σ by

$$R_s^{\text{normal}} = \sqrt{\frac{\omega\mu_0}{2\sigma}}.$$

For copper at 300 K, $\sigma = 5.8\times10^7\ (\Omega\cdot\text{m})^{-1}$ and at 1.3 GHz, $R_{s\,\text{copper}} = 9.4$ mΩ.

In superconductors the situation is very different. Below the transition temperature electrons of opposite spin pair up to form Cooper pairs spread over macroscopic dimensions of the order of the coherence length. Niobium has a coherence length of several 10 nm. Since Cooper pairs have a net spin zero they can all occupy the same ground state. It can be shown that this ground state is seperated from the energy levels of unpaired (normal-conducting) electrons by an energy gap. Cooper pairs can flow without resistance because collisions with defects are insufficient to scatter them into an unpaired state. But at finite temperature a thermal excitation of Cooper pairs is possible, the probability increasing with temperature as given by the Boltzmann factor. Hence the number of unpaired electrons drops exponentially below T_c. Frequently one therefore talks of a “two-fluid” model of superconductivity where Cooper pairs and normal electrons coexist at finite temperature.

This model also explains why superconductors have zero DC resistance. Since Cooper pairs flow without resistance they short out an applied field, and the (resistive) unpaired electrons are never accelerated. This situtation can be modelled by a resistor that is shorted out by a perfectly conducting wire in parallel.

Given the above expression for the surface resistance of normal conductors, one might be tempted to assume that SRF cavities also have zero surface resistance. However this is not the case: While Cooper pairs flow without resistance they do posess inertia that prevents them from instantaneously following a time-varying field. A rapidly oscillating field thus is not shorted out perfectly, allowing normal electrons to be accelerated and hence generate losses.

This situation can be modelled by an inductance (whose current lags the applied voltage but has no DC resistivity) in parallel with a resistor which represents the DC resistivity of the unpaired electrons, as shown in Figure 6. The total current in the two fluid model is calculated by adding the impedances in parallel. One can thus calculate the frequency and temperature scaling of the RF surface resistance of superconductors.

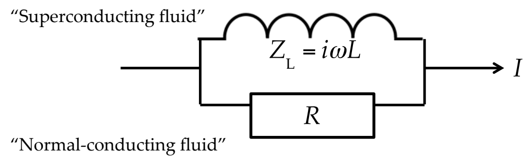

Figure 6: Circuit representation of a two-fluid superconductor.

From the free electron model we know that the resistivity of the normal conducting fluid scales as

$$\rho = \frac{m}{n_n e^2 \tau}$$

where n_n is the density of unpaired electrons of mass m and charge e that travel an average time τ before they are scattered. The impedance of the entire circuit in the limit that $R >> \omega L$ is then

$$Z = \frac{\omega L}{R}(\omega L + iR)$$

and the power dissipation is

$$P_{\text{diss}} = \text{Re}\left\{\frac{ZI^2}{2}\right\} = \frac{\omega^2 L^2}{2R} I^2.$$

The resistance of the normal conducting “fluid” R is inversely proportional to the density of normal-conducting electrons as given by the resistivity of the free-electron model. This drops exponentially according to the Boltzmann factor with decreasing temperature. Hence one immediately sees that the surface resistance of two-fluid superconductors scales as

$$R_s \propto \omega^2 \exp\left(-\frac{\Delta}{k_b T}\right)$$

where Δ is the material dependent energy gap between the Cooper-pair ground state and the thermally excited normal-conducting-electron states. The above expression is a simplified representation of the BCS resistance. A more complete expression can be derived from the BCS theory of superconductvity first formulated by Bardeen, Cooper and Schriefer (1957). The detailed derivation is complicated, but for niobium one finds that below $T_c/2$ the surface resistance is approximated by an expression that reflects the general scaling derived above:

$$R_{\mathrm{BCS}} \propto 8.9 \times 10^4 \ \frac{\mathrm{n\Omega \cdot K}}{\mathrm{GHz}^2} \frac{f^2}{T} \exp\left(-\frac{17.67\ \mathrm{K}}{T}\right)$$

where f is the frequency, T is the bath temperature, $T_c = 9.3$ K for niobium, so this approximation is valid for $T < 4.65$ K. At typical cavity operating at 1.5 GHz and cooled to 1.8 K will thus have a surface resistance in the low nΩ range.

Note that for superconductors, the BCS resistance increases quadratically with frequency whereas for normal conductors the surface resistance increases as the square root of frequency. For this reason, the majority of superconducting cavity applications favor lower frequencies (< 3 GHz) and normal-conducting cavity applications favor higher frequencies (> 0.5 GHz) there being some overlap depending on the application. Importantly, since SRF cavities favor low frequency they tend to disrupt the beams being accelerated much less (lower cavity impedance) since the walls are further from the beam. This is one of the main advantages of SRF for high current applications, where the beam disruption by the cavity can be severe. We will touch on this subject below.

The above expression represents the theoretical lower limit on the surface resistance (BCS resistance). However, measurements have shown that often the surface resistance also has a temperature independent term that is accounted for by the *residual resistance* R_{res}.

$$R_{\mathrm{s}} = R_{\mathrm{BCS}}(T) + R_{\mathrm{res}}$$

While not yet fully understood, it is known that the residual resistance in part arises from several sources, such as material defects, hydrides that can form on the surface due to hot chemical etching of the cavity and slow cool-down. One of the quantifiable residual resistance contributions is due to an external magnetic field being pinned in the superconductor. The pinned fluxon cores create small normal-conducting regions in the niobium that can be summed to estimate their net resistance. For niobium, the magnetic field contribution to R_{Φ} can be approximated by

$$R_{\Phi} \approx B_{\mathrm{ext}} \sqrt{f} \times 3.5 \frac{\mathrm{n\Omega}}{\mathrm{\mu T}\sqrt{\mathrm{GHz}}}$$

where B_{ext} is the external magnetic flux density present during the superconducting transition. This expression assumes that all the externally applied flux is pinned. While this appears to be the case with polycrystalline niobium, cavities made of large-grain niobium that have been heat treated may be less efficient at trapping flux. In that case the contribution to the above expression will be lower.

The Earth's nominal magnetic flux of 50 μT would then produce a residual surface resistance in a superconductor that is orders of magnitude greater than the BCS resistance, rendering the superconductor too lossy for practical use. For this reason, superconducting cavities are surrounded by magnetic shielding to reduce the field permeating the cavity to typically less than a few 1 μT.

Using the above approximations for a niobium a SRF cavity at 1.8 K, 1.3 GHz, and assuming a magnetic field of 1μT, the surface resistance components would be $R_{BCS} = 4.55$ nΩ and, $R_{\mathrm{res}} = R_{\Phi} = 3.2$ nΩ, giving a net surface resistance $R_{\mathrm{s}} = 8.6$ nΩ. If for this cavity $G = 270\ \Omega$ then the ideal quality factor would be $Q_o = 3.2 \times 10^{10}$.

In general, much care and attention to detail must be exercised in the experimental setup of SRF cavities so that there is no Q_0 degradation due to RF losses in ancillary components, such as stainless steel vacuum flanges that are too close to the cavity's evanescent fields. However, careful SRF cavity preparation, magnetic shielding and experimental configuration have achieved the ideal Q_o not only for low field amplitudes, also at high fields as shown in Figure 7.

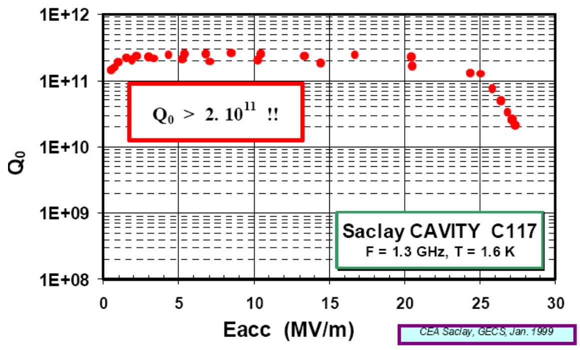

Figure 7: A nearly perfect superconducting cavity prepared at CEA-Saclay. At fields above 25 MV/m effects such as field emission cause the quality factor to deteriorate.

But few cavities make it to the magnetic field quench limit since small foreign particles on the surface heat the surrounding niobium which eventually exceeds the superconducting critical temperature, resulting in a thermal quench of the cavity. In other cases, particulates only a few microns in size can cause electron field emission, a quantum mechanical effect by which electrons in the cavity wall can literally tunnel out of the wall under the action of the electric field. The electrons are then accelerated, thereby draining energy from the field, which manifests itself as additional power dissipation. The quality factor hence drops as shown in Figure 7 above 20 MV/m. Eventually, many of the electrons collide elsewhere with the cavity wall, producing bremsstrahlung X rays and heat that may also quench the cavity.

Q vs *E*

When using superconducting RF cavities in particle accelerators, the field level in the cavity should generally be as high as possible to most efficiently accelerate the beam passing through it. The Q_0 values described by the above calculations tend to degrade as the fields increase, which is plotted for a given cavity as a "Q vs E" curve, where "E" refers to the accelerating electric field of the TM_{010} mode. Ideally, the cavity Q_0 would remain constant as the accelerating field is increased all the way up to the point of a magnetic quench field, as indicated by the "ideal" dashed line in Figure 8. In reality, though, even a well prepared niobium cavity will have a Q vs E curve that lies beneath the ideal, as shown by the "good cavity" curve in the plot.

There are many phenomena that can occur in an SRF cavity to degrade its Q vs E performance, such as impurities in the niobium, hydrogen contamination due to aggressive chemical etching during the cavity preparation, and a rough surface finish. At higher fields, typically, electron field emission or thermal breakdown due to particles cause problems.

After a couple decades of development, a necessary prescription for successful SRF cavity production is emerging. This includes:

- Eddy-current scanning for impurities of the raw niobium sheets used for cavity production,
- Good quality control of electron beam welding parameters used for cavity production
- Chemically etch the surface of the cavity to remove layers that were damaged during production. Maintain a low acid temperature to avoid contaminating the niobium with hydrogen.
- Electropolish of the cavity interior to achieve a very smooth surface.
- Bake the cavity in a vacuum furnace between 600 °C and 900 °C to remove hydrogen that has been dissolved in the niobium.
- High pressure rinse (HPR) of the cavity interior in a clean room with filtered water to remove particulate contamination that otherwise can cause field emission and thermal breakdown
- Carefully assembly of the cavity to other vacuum apparatus in a class 100 or better clean room with clean practices.
- Vacuum bake the cavity at 120 °C for about 48 hours.

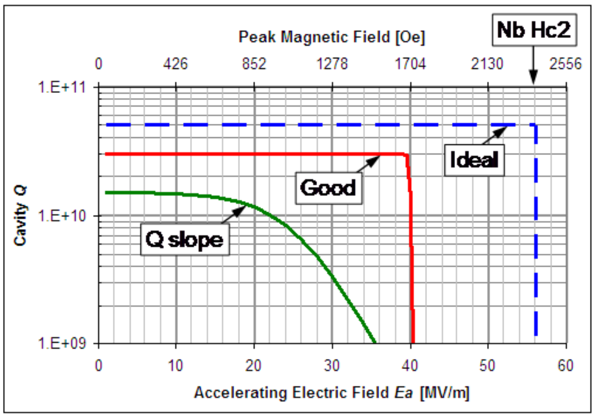

Figure 8: Example plots of SRF cavity Q_0 vs. the accelerating electric field E_a and peak magnetic field of the TM_{010} mode.

There remains some uncertainty as to the root cause of why some of these steps lead to success, such as the electropolishing and 120 °C vacuum bake. However, if this prescription is not followed, the Q vs E curve often shows an excessive degradation of Q_0 with increasing field, as shown by the "Q slope" curve in the plot. Finding the root causes of Q slope phenomena is the subject of ongoing fundamental SRF research. The insight gained could lead to simpler cavity fabrication processes as well as benefit future material development efforts to find higher T_c alternatives to niobium.

Wakefields and higher order modes (HOMs)

Superconducting cavities dissipate about 10^5 to 10^6 times less power than if they were made of copper. While this fact alone favors the use of SRF systems, it is its impact on the cavity design and that has proven to be the most attractive feature of SRF.

Power dissipation in copper cavities is so severe, that the geometry must be optimized to reduce this, often making very painful compromises with respect to the beam-cavity interaction. As a charged particle beam passes through a cavity, its electromagnetic radiation field is perturbed by the sudden increase of the conducting wall diameter in the transition from the small-diameter beampipe to the large hollow RF cavity. A portion of the particle's radiation field is then "clipped off" upon re-entrance into the beampipe and left behind as wakefields in the cavity. Mathematically, one can decompose the wakefields into an infinite spectrum of orthogonal cavity modes which are excited. These higher order modes (HOMs) are simply superimposed upon the externally driven accelerating fields in the cavity. The spawning of HOMs from the passing beam is analogous to a hammer striking a bell exciting many resonant mechanical modes.

The beam wakefields in an RF cavity represent a subset of the spectrum of the many electromagnetic modes, including the externally driven TM_{010} mode, which interact with the beam. In the best case, the beam quality suffers only a little. In the worst case a host of beam instabilities can occur that actually lead to beam loss, thereby limiting the maximum current that can be accelerated.

For a particle bunch with charge q, a length much shorter than the wavelength of a given cavity mode, and traversing the cavity at time t = 0, the amplitude of the wakefield voltage left behind in the cavity in a given mode is given by

$$V_{\text{wake}} = \frac{R}{Q}\frac{\omega}{2}q = kq$$

where R is the shunt impedance of the mode with frequency ω. The parameter k is referred to as the loss factor of the mode. The shunt impedance can be calculated from the solution of the electromagnetic fields of a mode, typically by a computer program that solves for the fields. We see that the excitation of the wakes is worse for high frequency cavities and if the mode's shunt impedence is high (i.e., when a mode's efficiency to accelerate charges is high). The efficiency increases when the beam tubes that allow charges to enter the cavity are small. Intuitively this makes sense since a cavity with small beam apertures concentrates the electric field on axis and has high R/Q_0, but also clips off more of the particle bunch's radiation field as deleterious wakefields.

To limit the wakefield excitation one therefore wants to use cavities with (a) low resonant frequencies, (b) large beam tubes and (c) shapes that minimize the HOM excitation (low shunt impedance). In addition, one must consider the build-up of HOMs from subsequent bunches due to superposition of the HOMs by the individual bunches. This calculation can be complex and depends strongly on the specific accelerator mode of operation. But it is worst when the HOM frequency is an integer multiple of the bunch repetition frequency. To safeguard against excessive resonant excitation, one therefore ensures that the HOMs can easily propagate out of the cavity and that they are damped so heavily that the HOM excited by a bunch decays rapidly.

HOM damping can be implemented by having loop antennas located at apertures on the side of the beampipe, with coaxial lines routing the RF to outside of the cryostat to standard RF loads. Another approach is to place the HOM loads directly on the beampipe as hollow cylinders with RF lossy material attached to the interior surface, as shown in Figure 9. This "beamline load" approach can be more technically challenging, since the load must absorb high RF power while preserving a high-vacuum beamline environment in close proximity to a contamination-sensitive SRF cavity. Further, such loads must sometimes operate at cryogenic temperatures (e.g., 80 K) to avoid large thermal gradients along the beampipe from the cold SRF cavity. The benefit of the beamline HOM load configuration, however, is a greater absorptive bandwidth and HOM attenuation as compared to antenna coupling. This benefit can be the difference between a stable vs. an unstable particle beam for high current accelerators.

Other options for HOM damping include waveguides that are attached to the beam tubes. The dimensions are chosen such that the accelerating mode does not propagate along the waveguide and hence is not damped. The HOMs can then be shunted via the waveguides to an HOM absorber far away from the cavity and which operates at higher temperature. Such a scheme is currently being investigated by HZB.

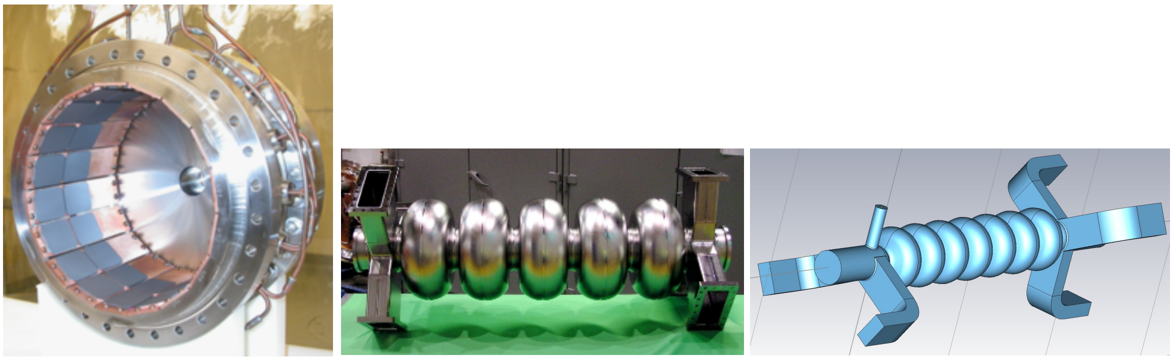

Figure 9: (left) Photograph of the Cornell electron storage ring beamline HOM load. Ferrite tiles (gray) are soldered to water-cooled elkonite plates that line the beam tube. HOMs that reach the load are quickly heavily damped by the ferrite. (middle) A prototype waveguide damped cavity developed by Jefferson Laboratory. (right) RF model used by HZB to simulate waveguide damping of an SRF cavity for BERLinPro.

At some threshold current, though, HOMs will always cause beam instabilities. One important goal of cavity design is thus to push this limit above the design current of the accelerator application. And this is where SRF systems play out their trump card. As mentioned above, copper cavities dissipate so much power that they invariably must, as the highest priority, maximize the shunt impedance, use small beam tubes and high frequencies. All of these design criteria are very detrimental for HOM excitation. Superconducting cavities, on the other hand, dissipate so little power that one can easily relax these constraints and in turn optimize the cavity design and frequency to drastically reduce the HOM power dissipation.

As an example, consider the copper cavity design shown in Fig. 10 that was used in the Cornell Electron Storage Ring (CESR) well into the 1990's. It had a high shunt impedance and small beam tube to limit the power dissipation. The excitation of HOMs was severe so that the loss factor for one cell was high. At the same time HOM extraction through the small beam tubes was difficult and the danger of resonant HOM excitation from one bunch to the next was great. Still, altogether 20 cells were required to provide the required total accelerating voltage so always five cells were grouped together in one cavity, raising the danger of trapped modes at the center of the cavity. The total loss factor (sum over HOMs) of all four cavities was 6.8 V/pC.

In the mid 1990's Cornell started replacing these cavities by superconducting systems. Power dissipation was no longer so important and one changed the geometry and increased the beam tube diameter, taking the "hit" in R/Q of the accelerating mode, which was nearly 3 times lower. Still, the power dissipation was low enough that it was not a major concern. In return, though, the loss factor of the HOMs per cell dropped by more than a factor two. In addition, the significantly larger beam tube allowed the HOMs to propagate more readily, so that very heavy damping coupld implemented to reduce the danger of resonant excitation. Dispite the low R/Q the SRF cavities, on account of their low power dissipation, can be operated at higher voltage and so only four single-cell cavities are required in CESR. Thus no trapped modes exist and the total loss factor for all for cavities was more than a factor 10 lower than the original copper RF system. By the time the complete SRF system was installed in CESR the stored current could be increased from 300 mA to over 700 mA. Together with some other modifications, the luminosity thus was raised by over a factor of five.

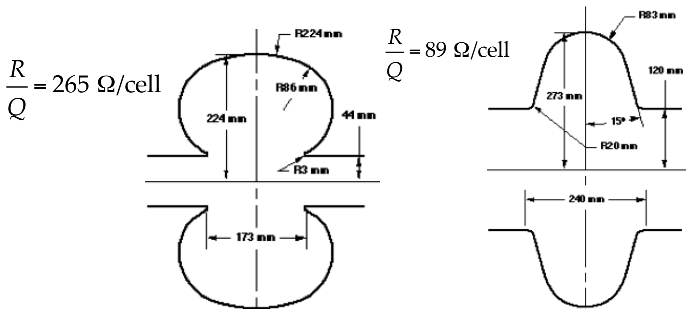

Figure 10: Comparison of the original copper RF system for the CESR Ring with the modern SRF system. Both operated at 500 MHz. To provide the full voltage required by CESR 20 copper cells were need (four 5-cell cavities). When these were replaced by the niobium system, four single-cell cavities were sufficient.

To sum things up, the advantage of SRF systems lies in the fact that power dissipation is, unlike for copper systems, no longer a prohibative constraint that drives the design considerations. Instead, SRF units can be tailored for a given accelerator application. This invaluable flexibility is especially useful when designing units for long-pulse or CW high current applications or machines requiring exceptional beam quality.

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